To Mr. JOHN WILSON TEACHER of MATHEMATICKS in *Edinburgh*,

Ingenious SIR,

WE have peruſed your Treatiſe of ARITHMETICK with Pleaſure; and, as we have always been of Opinion, That, by your Profeſſion and Practice, you have been very uſeful in *teaching* ſeveral Parts of the MATHEMATICKS; ſo we are convinced, that by this little Treatiſe you have given a publick Teſtimony of your ſuperior Capacity to teach the Science, of which you appear to be ſo much a Maſter. The whole is laid down in a conciſe, rational and intelligible Manner, in our Opinion, inferior to none extant: But particularly the DECIMAL FRACTIONS are treated of, ſo fully and accurately, that every Man of Buſineſs muſt be engaged to ſtudy that Part, which you have now rendred ſo plain, that it muſt undoubtedly facilitate their Practice.

YOU have been very ſucceſsful in making up a Deficiency, which appears in moſt of the Books formerly publiſhed upon that Subject; by giving Demonſtration and Reaſon for the *Operations* of every *Rule*, without which no *Arithmetical* Queſtion can ever be reſolved with Certainty: And, as you have thus laid down a ſure Foundation, upon which the more ordinary TEACHERS may now proceed, we think it a Duty we owe to you and our Country, to give this Opinion of it. We wiſh all happy Succeſs to you and your Labours, and are,

Dear SIR,

Your Sincere Friends,

Edinburgh,
March 23, 1742.

G. DRUMMOND.
D. SPENCE.
FRANCIS RUSSELL.

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Dear SIR,

Your Sincere Friends,

Edinburgh, G. DRUMMOND.
March 23, 1742. D. SPENCE.
FRANCIS RUSSELL.

AN INTRODUCTION TO Arithmetick,

CONTAINING THE
ARITHMETICK of INTEGERS,
of VULGAR FRACTIONS, and
of DECIMALS *Finite*, *Infinite*
Repeting, and *Infinite Circulating.*

Together With
The more neceſſary and uſeful *Rules* in Buſineſs, *viz.* The *Rules* of *Three*, of *Five*,
of *Fellowſhip*, and *Rules* for *Practice.*

To the whole is ſubjoined

An APPENDIX,

Shewing the common Practice of Meaſuring
MASON *and* JOYNER-WORK.

By JOHN WILSON *A. L M.* Teacher of MATHEMATICKS, and conſequently, NAVIGATION. Alſo ARITHMETICK ſeparately, at particular Hours.

EDINBURGH:
Printed by R. DRUMMOND and Company, at their Printing-houſe in *Swan-Cloſe*, for the AUTHOR. MDCCXLI.

Entred in STATIONERS HALL, and no Copies Authentick, but ſuch as are ſubſcrib'd by the AUTHOR, thus,

---

TO
GEORGE SKENE
OF
That ILK, Esquire,

SIR,

GRATITUDE, which diſtinguiſhes the great and generous Soul from the mean and low Spirit, as much as Reaſon does Human nature from the Brutal kind; Gratitude, (I ſay) obliges me to take this opportunity of making a publick acknowledgement of the many civilities, and kind offices, you have, upon all occaſions, loaded me with, from the firſt commencement of our acquaintance to this day.

WHAT inhances their value, and heightens my grateful reſentment of them, is, that I look on them as the real product, and genuine offspring of ſincere friendſhip, that ſacred tye and cement amongſt the ſocial part of mankind, which makes a harmony on Earth, reſembling that which univerſally prevails amongſt the Bleſſed above.

IT is in confidence of this friendſhip, Sir, that I preſume to preſent to you this ſmall Treatiſe of ARITHMETICK, not doubting of your favourable protection and patronage.

YOUR skill and knowledge in the *Mathematical Sciences* entitle you to it; and the ſubject ſpeaks for itſelf; ARITHMETICK being the channel, by which the laſt reſults of thouſands of beautiful and uſeful *Mathematical Problems* are conveyed to thouſands of mankind, who, by its means, reap the benefit of them, tho' they have not acceſs to the ſource and fountain from whence they flow.

HERE, Sir, I cou'd, after the example of other Authors, inſiſt upon your Character, and draw it in the brighteſt colours, without doing it to the advantage.

THIS, Sir, I cou'd do with the outmoſt candour and ingenuity on my part, and with the univerſal applauſe of all that have the happineſs of your acquaintance; but, as I know your modeſty, I chuſe rather, by abſtaining from it, to ſave you a Bluſh, and my ſelf the unjuſt imputation of Flattery, amongſt thoſe who have not had that happineſs, which has afforded ſo much pleaſure and ſatisfaction to,

SIR,

Your moſt Obliged,

and moſt Obedient,

Humble Servant,

JO. WILSON.

THE

PREFACE.

T*HAT* ARITMETICK *is come to a greater Perfection in this than in any preceeding Age, I believe will be readily granted by every Body. Our printed Courſes of it are handled by different Authors with the greateſt Accuracy and Exactneſs, and contain a vaſt Number of Rules in themſelves excellent and uſeful; but this Multiplicity of them renders the Bounds and Limits of a Syſtem of* Arithmetick *to ſome very undeterminate and uncertain; wherefore it will not be amiſs to determine its Limits. Previous to which, I muſt obſerve, That* FRACTIONS *are* NUMBERS, *for 'tis plain, that* 8 *Pence* (or $\frac{8}{240}$ *of a Pound) is a* Number, 1 *Penny (or* $\frac{1}{240}$ *of a*

Pound) being conſidered as Unity. We ſhall begin then with the Definition *of* Arithmetick.

ARITHMETICK *is the Science of* Numbers, *and the Art of Computation by them.*

As a Science *it muſt demonſtrate its own* Rules, *and as an* Art *it muſt put them in* Practice.

'Tis plain, that Number *is the Object of* Arithmetick.

Abſtracting from the Notation *of* Numbers, *the* Rules *of* Arithmetick *are but Four, ariſing from four Queſtions, which are obvious, and may occur to any who has but once heard of* Numbers. 1. *What is the Sum of two or more Numbers taken together? Hence we have* Addition *of* Numbers. 2. *What is the Difference of two given* Numbers? *Hence* Subtraction. 3. *What is the amount of a given* Number *as often taken as another given* Number *contains* Unity? *Hence* Multiplication. 4. *How often is one given* Number *contained in another? Hence* Diviſion. *Theſe four Rules, viz.* Addition, Subtraction, Multiplication,

plication, *and* Diviſion, *I call the* Algorithm, *or* Arithmetick *of* Numbers.

Now, as we diſtinguiſh Numbers *into* Integers *and* Fractions, Fractions *into* Vulgar *and* Decimal, *and* Decimals *into* Finite, Infinite Repeting, *and* Infinite Circulating; *we have a five-fold* Arithmetick *of* Numbers, *viz. The* Arithmetick *of* Integers, *of* Vulgar Fractions, *of* Finite Decimals, *of* Infinite repeting Decimals, *and of* Infinite circulating Decimals, *and theſe five make a complete Syſtem of* Arithmetick, *taking* Arithmetick *in a ſtrict and rigorous Senſe, as the Science of* Numbers.

This Syſtem of Arithmetick, *if well underſtood, is like a fine Inſtrument in the hands of a skilful Artiſt, by which he can ſurmount the greateſt difficulties in other Arts, but ſtill the direction of the Inſtrument, in working, depends on theſe Arts in which it is employed. For inſtance, by the Rules of* Arithmetick, *we can find the* Longitudes, Latitudes, Right Aſcenſions, Declinations, &c. *of the heavenly Bodies, but ſtill the direction for the application of theſe Rules depends upon the* *principles of* Aſtronomy *and* Spheric Trigonometry.

The Extraction *of* Roots, *is no part of* Arithmetick, *as not depending entirely on its firſt Elements and Principles; for inſtance, we can neither diſcover, nor underſtand the methods of* extracting *the* Square-root, *without having recourſe either to the 4th* Prop. *of the 2d Book of* Eucl. Ele. *or to an* Algebraic Square *raiſed from a* Binomial Root.

The other Rules, as thoſe of Three, Fellowſhip, Falſe, *&c. which are found in all our printed Courſes of* Arithmetick, *and are, by ſome, miſtaken for* Arithmetical Rules, *are indeed not ſo, in ſo far as they depend upon the Doctrine of* Proportion, *which owes its origin to* Geometry, *and not to* Arithmetick : *For, as* Number *is the object of* Arithmetick, *ſo* Magnitude *is the object of* Geometry; *and 'tis plain from the 3d and 4th Definitions of* Euclid'*s 5th* Element *of* Geometry, *that the* ratio *of the quantities of the ſame kind, is the mutual habitude, reſpect or relation the one quantity bears to the other, as to their* Magnitude; *and* Proportion *is the ſimilitude of* ratios, *ſo* Proportion *is originally* Geometrical. *And hence it is, that, in accounting for the Method of finding a fourth* Proportional *to three given* Numbers, *we are obliged to have recourſe to the 16th* Prop. *of* Eucl. *6th* Element, *where he demonſtrates, that if four Terms are proportional, the* Rectangle *or* Product *of the extremes, is equal to that of the middle Terms.*

My principal Intention in writing theſe Sheets was, to lay down a rational Syſtem of Arithmetick, *as ſuccinct, as poſſible, and, at the ſame time, ſo plain and clear, that the meaneſt capacity might eaſily comprehend it. How far I have ſucceeded in my Deſign, I muſt leave to the judgment of Mankind; mean time I beg the Reader's good nature may excuſe any eſcapes my weakneſs or inadvertency may have occaſioned. I hope the old obſerve,* Humanum eſt errare, *may apologize for my failings, and prevail with the Reader to pardon them.*

For the Practice of moſt caſes in the Arithmetick *of* Infinite repeting Decimals, *I am indebted to a ſmall Treatiſe of* Arithmetick, *written by our learned countryman, the Reverend Mr.* George Brown, *Miniſter of the Goſpel. He advances*

vances no theory of his Doctrine. That, with the few caſes in which he is deficient, I have here ſupplied. I do not pretend to make any boaſt of it, having only walked a few ſteps further in the road he chalkt out for me.

Seeing Finite Decimals *have an* Algorithm *of their own, which is full and complete, as being the ſame with that of* Integers, *due regard being had to the place of the* Decimal *point; likewiſe* Infinite repeting Decimals *have their* Algorithm *fully as complete, and yet* Infinite Circulates *can be wrought by neither of theſe* Algorithms, *we have all the reaſon in the world to conclude, that they too have an* Arithmetick *of their own. This* Arithmetick *of* Infinite Circulates *I have laid down as methodically and diſtinctly as I could. If the Reader thinks I might have done it in fewer words, all the apology I can make is, that this is my firſt attempt that way. The* Operations *in* Multiplication *and* Diviſion *are tedious, I muſt own; but then they are, to the laſt exactneſs, juſt.*

The Doctrine of Proportion, *upon which the other Rules which are not purely* Arithmetical *depend,*

depend, I have ſufficiently illuſtrated in the Rules of Three, Five *and* Fellowſhip, *theſe being the moſt conſiderable and uſeful in Buſineſs, the reſt I have neglected*; 1ſt, *As being plain and eaſy to any that is tolerably exerciſed in* Proportions. 2dly, *Leſt they ſhould ſwell this piece to a greater volume than was intended, (for I deſigned it for the pocket.) And*, 3dly, *becauſe they are laid down ſo fully by other hands, that my repetition of them would be but a* crambe repetita.

The Rules for Practice *I've laid down, becauſe they depend upon principles purely* Arithmetical, *viz. the aliquot parts of a pound, which are found by ſimple* Diviſion, *and likewiſe becauſe they are of great uſe in Buſineſs, and in many caſes very expeditious; yet I could rather chooſe to work* Decimally, *as being moſt expeditious in all caſes whatſoever. I know it is objected by ſome, That they loſe by* Decimals. *But if the Practitioner would continue the* Decimal *of the price till it is limited, he'll find he loſes nothing.*

Propor-

Proportions of *Scots* and *Engliſh* Meaſures.

THE *Scots*, *Paris*, and *Amſterdam* Ounces, are to the *Engliſh* Troy Ounce, as 95 to 96.

The *Scots*, *Paris*, or *Amſterdam* Grain, is to the *Engliſh* Troy Grain, as 19 to $23\frac{1}{25}$

The *Scots* Pound, is to the *Engliſh* Averdupois Pound, as 38 to 35.

The *Scots* Firlot Wheat Meaſure, is to the *Engliſh Wincheſter* Buſhel, as 100000 to 99042.

The Wheat Firlot, is to the Bear Firlot, as $21\frac{1}{4}$ to 31.

The *Scots* Ell, is to the *Engliſh* Yard, as $3\frac{1}{18}$ to 3.

The old *Scots* Foot, is to the *Engliſh* Foot, as 186 to 185.

The *Scots* Pint contains 103.421 Cubic Inches.

The Reader is deſired to correct the following Errors, which have eſcaped the Preſs, before he begins to read.

Page 32. *Ex.* of *Diviſion*, 3d place from the left hand of the *Dividend*, for 9 read 6. P. 34. l. 4 from the foot, for *is* r. *in.* P. 35. 2d Parag. l. 1. for 288. r. 228. P. 38. *Ex.* 4. for 88 r. 80. *Ibid.* l. 3. and l. 7 from the foot, for *right hand* r. *left hand*, &c. P. 71. l. *ult.* above 798. for *m.* r. *Pr.* P. 72 for 74. r. 72. P. 91. l. 10. r. *of the gain*, &c. P. 94. in the *Diviſor* of the firſt *Diviſion*, inſtead of cuting off 80. cut off 0 only. P. 108. In the reaſon of *Rule* IV. for $\frac{4}{5}$ r. $\frac{4}{15}$, and for $\frac{12}{5}$ r. $\frac{12}{15}$. P. 116. in obſerve 1. l. 3. for *N. B.* r. *Coral.* *Ibid.* l. 6. for *N. B.* r. *Coral.* P. 118. in the Citation, &c. for *Coral.* r. *N. B.* P. 156. Parag. 3. l. 4. for *Shillings* r. *Shilling.* P. 224. after, as a proof, &c. l. 3. for *Simple* r. *Vulgar.* P. 226. l. 1 for *circalate* r. *circulate.*

CHARACTERS Explained.

$=$	Signifies	Equal.
$+$		Plus or Addition.
$-$		Minus or Subtraction.
$\times$		Multiplication, *into*, or multiplied into.
$\frac{12}{3}$		12 divided by 3, or thus, 3) 12 (

In *Example* 2d, Page 240, I propoſe to divide 500.45,45 by 6.36,36 I mark it thus $\frac{500.45,45}{6.36,36}$ the mixt *Number* with the *Vulgar Fraction* anſwering to 500.45,45 is 500. $\frac{5}{11}$, and that anſwering to 6.36,36 is 6 $\frac{4}{11}$, their *Diviſion* I mark thus, 6 $\frac{4}{11}$) 500 $\frac{5}{11}$ (and reducing theſe mixt *Numbers* to improper *Fractions*, I have $6\frac{4}{11} = \frac{70}{11}$ and $500\frac{5}{11} = \frac{5505}{11}$ their *Diviſion* ſtands thus, $\frac{70}{11}$) $\frac{5505}{11}$ (and actually dividing them, their quot is $\frac{5505}{70}$, likewiſe dividing the *Decimals* firſt given, their quot is 78.6,428571; ſo the whole progreſs ſtands thus, $\frac{500.45\ 45}{6.36,36} = 6\frac{4}{11}$) $500\frac{5}{11}$ ($= \frac{70}{11}$) $\frac{5505}{11}$ ($= \frac{5505}{70}$) $= 78.6,428571$.

INTRODUCTION TO Arithmetick.

NOTATION of NUMBERS.

One,	Five,	Ten,	Fifty,	Hundred,	Five hundred,	Thouſand.
I,	V,	X,	L,	C,	D,	M.

ALL the Figures us'd by the Ancient *Romans* in *Arithmetick* were the above Seven, whoſe values are annex'd, and by various combinations of theſe, they expreſſed all their other Numbers according to the following Rules.

I. As often as any one Figure is repeted without interruption, ſo often its value is to be taken.

II. A Figure of a leſſer value on the left hand of a Figure of a greater value, takes the leſſer value from the greater.

III. A Figure of a leſſer value on the right hand of a Figure of a greater value, adds their values.

EXAMPLES.

Two,	Four,	Twenty,	Forty,	Ninety,	Six hundred.
II,	IV,	XX,	XL,	XC,	DC.
Three,	Six,	Thirty,	Sixty,	Two hundred,	Seven hundred.
III,	VI,	XXX,	LX,	CC,	DCC.
Seven,		Fifteen,	Seventy,	Hundred and Fifty.	
VII,		XV,	LXX,	C L.	
Nine.					
IX.					
Nineteen.					
XIX.					
Eighteen.					
XIIX.					

They put Iↄ for D, or 500, and by repeting ↄ, they made the Number ten times greater; thus,

Iↄ = 500,
Iↄↄ = 5000,
Iↄↄↄ = 50000.

The ſame way they put CIↄ for M, or 1000, and by repeting C and ↄ, they made the Number ten times greater; thus,

CIↄ = 1000,
CCIↄↄ = 10000,
CCCIↄↄↄ = 100000,
CCCCIↄↄↄↄ = 1000000.

They likewiſe expreſſed Thouſands by a *Line* over the head of ſmaller Numbers; thus,

$\overline{\text{III}}$ = 3000,
$\overline{\text{V}}$ = 5000,
$\overline{\text{X}}$ = 10000,
$\overline{\text{C}}$ = 100000,
$\overline{\text{M}}$ = 1000000,

N. B. In ſome Editions of Cæſar's Commentaries, *I find* ∞ put for CIↃ, or 1000.

---

0 1 2 3 4 5 6 7 8 9.

All the Figures uſed in *Arithmetick*, at this Day, are the above Ten; of which the firſt (0) called a Cypher, ſignifies nothing, and yet is of great uſe in *Arithmetick:* The next is called One, Unity, or an Unit; it is that by which every particular is called one of its kind; and is the Fountain of all Numbers, it-ſelf no Number; for,

Number is a Collection or Multitude of Units.

The next Figure (2) exceeds the preceeding (1) by one, and the following Figure (3) exceeds its preceeding Figure (2) by one, &c. So that taking the Figures in the natural order in which they ſtand above, each Figure exceeds the immediately preceeding by one, or, (which is the ſame thing) each Figure ſignifies as many Units as it has places from (0). This is the *ſimple value* of the Figures.

Beſides this ſimple value, the Figures have a *local value*, according to the place in which they ſtand, reckoning the places from the right hand towards the left; thus,

8th, *7th*, *6th*, *5th*, *4th*, *3d*, *2d*, *1ſt Place*, &c. · · · · · · · ·

A Figure in the firſt place has its ſimple value, a Figure in the ſecond place has ten times the value it would have, were it in the firſt; a Figure in the third place has ten times the value it would have, were it in the ſecond; and univerſally, *each place has ten times the value of that immediately preceeding it.*

From this conſideration of the *ſimple* and *local* values of the Figures, we can account for the *Addition*, *Subtraction*, *Multiplication*, and *Diviſion* of Numbers; and from the ſame conſideration we may find a method of reading and writing Numbers, *viz.* by aſſigning names to the ſeveral places; thus,

	8	X of Quadrillions.
Quadrillions	6	Millions of Trillions.
	8	C of M of Trillions.
	9	X of M of Trillions.
	7	M of Trillions.
	5	C of Trillions.
	6	X of Trillions.
Trillions	3	Millions of Billions.
	9	C of M of Billions.
	7	X of M of Billions.
	4	M of Billions.
	8	C of Billions.
	5	X of Billions.
Billions	7	Millions of Millions.
	2	C of M of Millions.
	9	X of M of Millions.
	6	M of Millions.
	4	C of Millions.
	7	X of Millions.
	3	Millions.
	8	C of Thouſands.
	5	X of Thouſands.
	9	Thouſands.
	6	Hundreds.
	5	Tens.
	8	Units.

RULE I.

RULE I. *To read any Number.*

To the ſimple value of each Figure joyn the name of its place, beginning at the left hand, and reading toward the right.

N. B. 1. Becauſe every third place is Hundreds, theſe may be marked with a point above, thus;

$$86\dot{8}97\dot{5}63\dot{9}74\dot{8}57\dot{2}96\dot{4}73\dot{8}59\dot{6}58$$

N. B. 2. Becauſe the Figure on the left hand of each ſecond Hundred place is *Millions*, *Billions*, *Trillions*, &c. theſe may be marked with a point below; thus,

$$8\underset{\cdot}{6}\dot{8}97\dot{5}6\underset{\cdot}{3}\dot{9}74\dot{8}5\underset{\cdot}{7}\dot{2}96\dot{4}7\underset{\cdot}{3}\dot{8}59\dot{6}58$$

N. B. 3. A Figure on the left hand of a Hundred point, having its ſelf no point, is always Thouſands.

N. B. 4. In the order of reading after Hundreds follow always Tens.

8	Eight, or eight Units.
08	No Tens, eight Units.
008	No Hundreds, no Tens, eight Units.

From the above Notation it is plain, that Cyphers on the left hand of a ſignificant Figure do not change its value.

8	Eight, or eight Units.
80	Eighty, or eight Tens, no Units.
800	Eight Hundreds, no Tens, no Units.

From the above Notation it is plain, that Cyphers on the right hand of a ſignificant Figure augment its value in a decimal proportion; ſo that the only uſe of Cyphers in the *Arithmetick* of *Integers* is to ſupply places, ſo as to augment the value of ſignificant Figures, by caſting them into higher places.

RULE II. *To write any Number.*

Write down the Figures in the ſame Order their values are expreſſed; beginning at the left hand, and writing toward the right.

So this Number (Seventy four thouſand, ſix hundred and fifty eight) is written thus, 74658.

N. B. If in pronouncing the Number, any places are omitted, theſe places muſt be ſupplied with Cyphers. So this Number (Six Millions, five thouſand and twenty eight) is written thus, 6005028.

DEFINITIONS.

1. An Intiger is any whole number, as 8, or 75.

2. A Digit is any one of the Nine ſignificant Figures; for 0 ſignifies *nothing*.

3. A Number is ſaid to *meaſure* another number, when it is contained in that other number a certain number of times, or when it divides that other number without a remainder.

4. An Even number is that which is meaſured by 2.

5. An Odd number is that which is not meaſured by 2, or that which differs from an even number by Unity.

6. A Prime number is that which Unity only meaſures, as 5, or 11.

7. A Compoſite number, is that which ſome number meaſures, as 6, which is meaſured both by 2, and by 3.

8. Numbers compoſite betwixt themſelves, are ſuch as have one common meaſure, as 6 and 9, which are both meaſured by 3.

ADDITION.

ADDITION is a Collecting or Gathering together of two or more numbers into one Sum.

The mark of *Addition* is (+) *plus*, as 5 + 3 = 8, that is, five *plus* three, or five having three added to it, is equal to eight.

The numbers are for ordinary written one below the other as in the margine,

	5	3
	3	5
Total	8	8

If I have 372 + 426, becauſe only ſimilar or like things can be collected into one Sum, I place the one Number under the other (as in the margine) like parts under like parts, *viz.* Units under Units, Tens under Tens, &c. Then I ſay, 6 Units and 2 Units make 8 Units, which I ſet below in the place of Units; then 2 Tens and 7 Tens make 9 Tens, which I ſet below in the place of Tens; then 4 Hundreds and 3 Hundreds make 7 Hundreds, which I ſet below in the place of Hundreds, ſo the whole Sum is 798. Hence,

372
426
———
798

RULE I.

RULE I. *In Addition Units only can ſtand in the place of Units, Tens only in the place of Tens, Hundreds only in the place of Hundreds,* &c.

If I have 768 + 959 + 824 + 738, I ſet them according to Rule 1*ſt*, Units under Units, &*c*. Then I ſay, 8 and 4 make 12, and 9 make 21, and 8 make 29, where I have 2 Tens, and 9 Units; theſe 9 Units I ſet below the place of Units, by *Rule* 1*ſt*, and becauſe the next place is ten times the value of this place of Units, I carry my 2 Tens into the next place ſimply as 2; and ſay, 2 Tens, which I collected out of my Units, and 3 Tens, make 5 Tens, and 2 make 7, and 5 make 12, and 6 make 18, where I have 1 Ten, and 8 over, theſe 8 I ſet below in the place of Tens, by *Rule* 1*ſt*; and, becauſe each place is ten times the value of that immediately preceeding, I carry my 1 Ten into the next place, as only one of that place, and ſay, 1, which I collected out of the preceeding place, and 7 make 8, and 8 make 16, and 9 make 25, and 7 make 32, where I have 3 Tens, and 2 over, theſe 2 I ſet below in their proper place, and the 3 Tens, which belong to the next place, I ſet on the left hand, there being no figure in the next place to which I can carry it, as ſimply 3. So the whole Sum is 3289, which affords me,

$$\begin{array}{r}768\\959\\824\\738\\\hline 3289\\\hline\end{array}$$

RULE II. *Carry always according to the value of the immediately following place.*

If I have Pounds, Shillings, Pence and Farthings to be added, I place the ſeveral Sums, as in the margine, Pounds under Pounds, Shillings under Shillings, &c. and in each of theſe Units, under Units, Tens under Tens, &c. by *Rule* 1ſt. Then, becauſe 1 Penny is in value 4 Farthings, in ſumming up my Farthings for every 4 I carry 1 to the place of Pennies, and, becauſe 1 Shilling is in value 12 Pennies, in ſumming up my Pennies, for every 12 Pennies I carry 1 to the place of Shillings: The ſame Way, becauſe 1 Pound is in value 20 Shillings, in ſumming up my Shillings, for every 20 I carry 1 to the place of Pounds by *Rule* 2d. So, beginning with my Farthings, I proceed thus, 2 Farthings and 3 Farthings make 5, and 3 make 8, and 2 make 10 Farthings; where I have 2 fours, or 2 Pence, and 2 Farthings; theſe 2 Farthings I ſet below in their proper place, and carry my 2 Pence to the next place, *viz.* that of Pence, and ſay, 2 Pence, that I collected out of my Farthings, and 1 make 3, and 1 make 4; here, tho' I have the Sum of my Units, I do not ſet them down below in their proper place, but along with them I ſum up my Tens; thus, 4 and 10 (for every 1 in the place of Tens is one Ten) make 14, and 10 make 24, and 10 make 34, and 10 make 44 Pence; here I have 3 Twelves, or 3 Shillings, and 8 Pence over; theſe 8 Pence I ſet below in their proper place, and

L.	*s.*	*d.*	*f.*
879	16	11	2
798	18	10	3
247	19	10	3
178	15	11	2
2105	11	08	2

carry my 3 Shillings to the place of Shillings; thus, 3 Shillings that I collected out of my Pence, and 5 Shillings make 8, and 9 make 17, and 8 make 25, and 6 make 31; here I have 3 Tens, and 1 over, which I ſet down in its proper place of Units, and carry my 3 Tens to the next place, being the place of Tens (for here I can ſum up my Tens by themſelves, becauſe juſt 2 Tens make 20 Shillings, which is 1 of the immediately following place, *viz.* 1 Pound) then I ſay, 3 Tens, which I collected out of my Units, and 1 Ten make 4 Tens, and 1 make 5, and 1 make 6, and 1 make 7 Tens; here I have 3 Twenties, and 1 Ten over, which I ſet down below in its proper place, and carry my 3 twenty Shillings, or 3 Pound to the place of Pounds, which, being *Integers*, I ſum up, as in the preceeding Example.

From the manner of working this third Example, follows

RULE III. *In adding* Integers *and their parts, (as Pounds, Shillings, Pence and Farthings) if in ſumming the parts you carry a juſt number of Tens; ſum up firſt your Units, and carry Tens to the next place, as being the place of Tens; then ſum up your Tens, and carry at the number of Tens contain'd in the value of the immediate following place, as in Shillings, where you carry at* 20, *which is juſt two Tens; after having ſummed up your Tens, for every* 2 *in that ſum*

ſum carry 1 *to the next place, which is the place of Pounds.*

If you do not carry at a juſt number of Tens, ſum up your Units and Tens at once, as in Pence, where you carry at 12, *which is more than one, and leſs than two Tens; and for every* 12 *in the ſum, you carry* 1 *to the next place, which is the place of Shillings.*

These three Rules extend to all the caſes that can fall out in *Addition.*

Example 2d, in addition of time, where

	Y.	*m.*	*w.*	*d.*	*h.*	*min.*
60 Minutes = 1 Hour,	78	11	3	6	18	56
24 Hours = 1 Day,	25	10	2	4	21	48
7 Days = 1 Week,	38	12	3	5	16	47
4 Weeks = 1 Month,	68	11	2	5	17	39
13 Months = 1 Year.						
	212	08	1	2	03	10

Here I begin with Minutes, and ſay, 9 and 7 make 16, and 8 make 24, and 6 make 30; I ſet 0 below, and carry my three Tens to the next place, being the place of Tens; then I ſay, 3 that I carried, and 3 make 6, and 4 make 10, and 4 make 14, and 5 make 19; and becauſe 60, or ſix Tens, make one Hour, for every 6 in this ſum of Tens, (19) I carry 1 to the next place of Hours; ſo I ſet down 1 below, and carry 3: Then I ſay, 3 that I carried, and 7 make 10, and 6 make 16, and 1 make 17, and 8 make 25, and 10 (for 1 in the place of Tens is one Ten) make 35, and 10 make 45, and 20 make 65, and 10 make 75; ſo the ſum of my Units and Tens, taken together, is 75, where I have three 24s, and 3 over, I ſet my 3 of Exceſs below, and carry my three 24s, as 3 Days, to the place of Days, and ſay, 3 that I carried, and 5 make 8, &c.

SUBTRACTION.

SUBTRACTION takes a leſſer Number from a greater, that their difference may appear.

The mark of *Subtraction* is [—] *minus*, as 8—3=5, that is 8 *minus* 3, or 8 having 3 ſubtracted from it, is equal to 5.

If I have 768—345, I place the leſſer Number below the greater, and becauſe only ſimilar or like Things can be ſubtracted, I place Units under Units, Tens under Tens, &c. Then I ſay, 5 Units from 8 Units, and 3 Units remains, which I ſet down below; 4 Tens from 6 Tens, and 2 Tens remain, which I ſet down below in the place of Tens; then 3 Hundreds from 7 Hundreds, 4 Hundreds remain: So the whole Difference, or Remainder is 423. Hence,

768	*Minuend*,
345	*Subtrahend*,
423	*Difference*.

RULE I. *In* Subtraction *Units only can ſtand in the place of Units, Tens only in the place of Tens, only Hundreds in the place of Hundreds*, &c.

Axiom. If to unequal numbers equal numbers are added, the difference of the wholes is the ſame with the difference of the unequal numbers firſt given.

8	8+10=18	8+12=20	8+24=32
3	3+10=13	3+12=15	3+24=27
5 *Difference*	5	5	5

If I have 824—368, I place the leſſer under the greater, Units under Units, Tens under Tens, *&c.* by Rule 1*ſt.* Then I ſay, 8 Units from 4 Units I cannot, but, becauſe Ten is the value of the following place, I add 10 to 4, and ſay, 8 from 14 and 6 remains, which I ſet down below, and add the Ten (which I added to the Units of my *Minuend*) as 1 to the Tens of my *Subtrahend*, (by *Axiom*) and ſay, 1 that I borrowed, and 6 make 7, from 2 I cannot, but, becauſe the following place is ten times the value of this place, I add 10 to 2, and ſay, 7 from 12, and 5 remains, which I ſet down below, and add the Ten as 1 to the place of Hundreds in my *Minuend*; thus, 1 that I borrowed, and 3 make 4, from 8, and 4 remains, ſo the whole difference is

824
368

456

is 456; and, becauſe I have added the ſame number to both *Minuend* and *Subtrahend*, the difference is the ſame as if no addition had been made by preceeding *Axiom:* Hence,

RULE II. *Borrow always according to the value of the immediately following place.*

If I have Pounds, Shillings, Pence and Farthings, as in the margin, I ſet them down Pounds under Pounds, Shillings under Shillings, &c. and in each of theſe columns Units under Units, Tens under Tens, &c. by *Rule* 1*ſt*. Then I ſay, 3 Farthings from 2 Farthings I cannot, but I borrow 4 (as being the value of the immediately following place) to the 2, and ſay, 3 from 6, and 3 remains, which I ſet down below, and carry the 4 Farthings (which I borrowed to my Farthings of the *Minuend*) as one Penny to the Pence of my *Subtrahend*, and ſay, 1 that I borrowed, and 10 make 11, from 5 I cannot, but to my 5 Pence I borrow 12, as being the value of the following place, and ſay, 12 and 5 make 17, ſo 11 from 17, and 6 remain, which I ſet down below, and add the 12 Pence (which I borrowed to the Pence of my *Minuend*) as 1 Shilling to my Shillings, and ſay, 1 Shilling that I borrowed, and 6 make 7, from 3 I cannot, but I borrow 10, (the next place

L.	*s.*	*d.*	*f.*
710	13	05	2
237	16	10	3
472	16	06	3

being the place of Tens) and ſay, 7 from 13, and 6 remains, which I ſet down below, and carries the 10 (which I borrowed to the Units of the Shilling in my *Minuend*) as one to the Tens of the *Subtrahend*, and ſay, 1 Ten that I borrowed, and 1 make 2, from 1 I cannot, but I borrow 2 Tens (which is 20, the value of the following place) to the 1 Ten in my *Minuend*, and 2 Tens from 3 Tens, and 1 remains, which I ſet down below, and carry the 2 Tens, or 20 Shillings (which I borrowed to the Tens of Shillings in my *Minuend*) as 1 Pound to the Units of Pounds in the *Subtrahend*, and ſay, 1 Pound that I borrowed, and 7 make 8, from 0 I cannot, but 8 from 10, and 2 remain, &c. Hence,

RULE III. *If you borrow a juſt number of Tens, ſubtract firſt your Units, (borrowing* 10 *if need be) then ſubtract your Tens, borrowing (if need be) the number of Tens contained in the value of the immediately following place, as in Shillings, where you borrow Twenty, which is juſt* 2 *Tens.*

But if you do not borrow a juſt number of Tens, ſubtract your Units and Tens at once, as in Pence, where you borrow 12, *which is more than one, and leſs than two Tens.*

Theſe three Rules extend to all the poſſible caſes in *Subtraction*; we ſhall illuſtrate them once more in an Example of *Subtraction* of time:

Here, becauſe 60 minutes are juſt 6 Tens, I ſubtract firſt my Units, then my Tens; thus, 6 from 3 I cannot, but I borrow 10, and ſay, 6 from 13, and 7 remains, which I ſet down below, and ſay, 1 Ten that I carried, and 3 make 4 Tens, from 2 Tens I cannot, but I borrow 6 Tens, or 1 Hour, and ſay, 4 Tens from 6+2, or 8 Tens, and 4 Tens remain, which I ſet down, and ſay, 1 Hour that I borrowed, and 17 make 18, from 15 I cannot, but I borrow 24 Hours, or one Day, and ſay, 18 from 24+15, or 39, and 21 remains, which I ſet down, &c.

Y.	*m.*	*w.*	*d.*	*h.*	*min.*
82	08	1	4	15	23
16	11	2	4	17	36
65	09	2	6	21	47

MULTIPLICATION.

IN MULTIPLICATION there are two numbers given, *viz.* one to be multiplied, called the *Multiplicand*, and another that multiplies it, called the *Multiplier*, and thoſe two go under the common name of *Factors*; the number produced by the multiplication of theſe two, is called the *Product*. Theſe terms being explained, the *Definition* is plain.

MULTIPLICATION is a taking, or placing of the *Multiplicand* as often as the *Multiplier* contains Unity.

Or, *Multiplication*, from a *multiplicand* and *multiplier* given, finds a third number, called the *product*, which contains the *multiplicand* as often as the *multiplier* contains unity.

The mark of *Multiplication* is [×] *into*, or *multiplied into*, as 3×4=12, that is 3 into 4, or 3 multiplied into 4 are equal to 12.

'Tis plain, from the operation in the margin, that it is a matter of indifference which of the two *factors* is *multiplicand*, and which is *multiplier*, for the *product* is the ſame in both caſes, and the eſſential property of *Multiplication* holds in both, *viz.* The *product* contains the *multiplicand* as often as the *multiplier* contains unity.

4 *multiplic.*	3	} *factors.*
3 *multiplier*	4	
12 *product*	12	

If the *multiplier* has cyphers on the right hand, multiply the ſignificant figure (or figures) in the *multiplier* into the *multiplicand*, and join the cyphers to the right hand of the *product*, and you have the product of the whole *Multiplication*; for the only uſe of cyphers in the arithmetick of *Integers* is to ſupply places..

4
3000
12000

It is plain the caſe is the ſame, if the *multiplicand* has cyphers on the right hand.

3000
4
12000

Conſequently, if both *multiplicand* and *multiplier* have cyphers on the right, to the right hand of the product of the ſignificant figures join the cyphers in both *factors*.

$$\begin{array}{r} 4000 \\ 300 \\ \hline 1200000 \\ \hline \end{array}$$

If your *multiplicand* conſiſt of ſeveral places, and your *multiplier* is a digit, ſet the *multiplier* under the Units of the *multiplicand*, and multiply it into each place of the *multiplicand*, firſt Units, then Tens, &c. carrying always according to the value of the immediately following place; thus, 8 times 6 make 48, I ſet down 8, and carry my 4 Tens to the next place; then, 8 times 9 make 72, and 4 that I carried, make 76, I ſet down 6 below, and carry 7 Tens to the next place; then, 8 times 7 make 56, and 7 that I carried make 63, which I ſet down, having no place further to which I can carry.

$$\begin{array}{r} 796 \\ 8 \\ \hline 6368 \\ \hline \end{array}$$

If your *multiplier* conſiſts of two, three, or more places, ſet it under the *multiplicand*, Units under Units, Tens under Tens, &c. Multiply firſt your Units into the whole *multiplicand*, (as before) then your Tens, then your Hundreds, &c. ſetting always the right hand figure of each *product* directly below the multiplying figure; the ſum of all theſe particular products is the product of the whole multiplication.

$$\begin{array}{r} 87968 \\ 78 \\ \hline 703744 \\ 615776 \\ \hline 6861504 \\ \hline \end{array}$$

N. B. In multiplying my 7 Tens into 8, the Units of my *multiplicand*, I ſay, 7 times 8 make

56,

56, I ſet down 6 under the multiplying figure, or under Tens of the *multiplicand*; for this 7 ſtanding in the place of Tens, is properly 70, and 70×8=560, and carrying 5 to the next place, and ſetting down 60, the cypher falls into the place of Units, and caſts the 6 into the place of Tens; ſo the cypher, making no alteration in the addition of the partial products, is juſtly neglected.

In the example in the margin, in multiplying 7 Hundreds into 8 Units of the *multiplicand*, I ſet my right hand figure of the *product* under Hundreds of the *multiplicand*; for here 7 is properly 700, &c.

857698
768

6861584
5146188
6003886

658712064

If you have cyphers intermix'd with ſignificant figures in the *multiplier*, multiply the ſignificant fiures only, ſetting the right hand figure of each product below the multiplying figure.

786804768
806009

7081242912
4720828608
6294438144

634171724250912

There are ſeveral compendious ways of working multiplication of *Integers*, which, being rather curious than neceſſary, I paſs over: Only one compend I ſhall here lay down, as being very uſeful in the arithmetick of infinite circulating *Decimals*. It is this, To multiply any number by a number conſiſting of any number of 9s, as 99, 999, &c. To the right hand of the *multiplicand* annex as many cyphers as your *multiplier* has 9s, and ſubtract the *multiplicand*.

8756×99=866844
875600
8756
866844

6876×999=6869124
6876000
6876
6869124

The reaſon is plain; for, 875600=8756×100, which contains 8756 juſt once more than 8756×99; if therefore, from 875600 you deduce 8756, you have 99 times 8756.

To multiply any number conſiſting entirely of 9s by itſelf,

Set 1 in the place of Units, then as many cyphers, except one, as there are 9s in the *multiplicand*, then 8, and on the left hand of 8 as many 9s as there are cyphers on its right hand; thus,

9900
99
99×99=9801 9801

999000
999
999×999=998001 998001

9999×9999=99980001

The reaſon is plain from the former caſe, and the operations annexed.

Definition 2. The component parts of any number are two, three, or more numbers, which multiplied together, produce that number; ſo 2 and 6 are the component parts of 12; for 2×6=12, and 2, 3 and 6, are component parts of 36, for 2×3=6, and 6×6=36, or 2×3×6=36.

Definition 3. The aliquot parts of any number are ſuch as meaſure that number; ſo 2, 3, 4, 6, 8 and 12, are the aliquot parts of 24; for any one of theſe meaſure 24.

Multiplication of Integers, *and their known parts by Integers.*

If your *multiplier* is a digit, ſet it under Units of the loweſt denomination, multiply it into all the parts, beginning at the loweſt, and carrying always according to the immediately following place.

EXAMPLE.

What is the price of eight bags of Meal, at 1 *l.* 2 *s.* 6 *d.* 2 *f. per* bag?

L.	*s.*	*d.*	*f.*
1	2	6	2
			8
9	0	4	0

Here I ſay, 8 times 2 make 16, which is juſt 4 Pence, and nothing over, ſo I ſet 0 down, and carry 4 to the place of pence; then I ſay, 8 times 6 Pence makes 48, and 4 that I carried makes 52, which is 4 twelves, or 4 Shillings, and 4 Pence, I ſet down the 4 Pence below, and carry 4 Shillings to the place of Shillings, ſaying, 8 times 2 Shillings make 16 Shillings, and 4 that I carried make 20 Shillings, which is 1 Pound, and nothing over, I ſet down 0, and carry my 1 Pound to the place of Pounds, ſaying, 8 times 1 Pound make 8 Pounds, and 1 that I carried make 9 Pounds, which I ſet down; ſo the anſwer is 9 *l.* 0 *s.* 4 *d.* 0 *f.*

If your *multiplier* is a compoſite number, reſolve it into its component parts, which muſt be all digits; multiply the ſum given by any one of theſe parts, and that product by the other; and ſo on till all your component parts are exhauſted, and you have the anſwer.

EXAMPLE. I.

What is the price of 72 Acres of ground, at 2 *l.* 8 *s.* 8 *d.* 2 *f. per* Acre?

8×9=72, that is, 8 and 9 are the component parts of 72.

So I firſt multiply the given ſum by 8; the *product* 19 *l.* 9 *s.* 8 *d.* I multiply by the other component part 9, and I have the whole anſwer, *viz.* 175 *l.* 7 *s.* 0 *d.* 0 *f.*

N. B. It is no matter which component part you multiply by firſt, which ſecond.

L.	*s.*	*d.*	*f.*
2	8	8	2
			8
19	9	8	0
		9	
175	7	0	0

EXAMPLE II.

What is the price of 144 Yards of cloth, at 1 *l.* 5 *s.* 10 *d.* 3 *f. per* Yard? 144=4×6×6.

Here I multiply the ſum given by 4, then I multiply that product 5 *l.* 3 *s.* 7 *d.* by 6. and that new product, 31 *l.* 1 *s.* 6 *d.* by my other component part 6, which give the anſwer to the queſtion, 186 *l.* 9 *s.* 0 *d.* 0 *f.*

L.	*s.*	*d.*	*f.*
1	5	10	3
			4
5	3	07	0
		6	
31	1	06	0
		6	
186	9	0	0

If your *multiplier* is not a compoſite number, whoſe component parts are digits, firſt multiply the ſum given by a compoſite number leſs than the *multiplier*, but the neareſt to it you can find, then multiply the given ſum by the difference of the *multiplier*, and that neareſt compoſite number, theſe two *products* give the anſwer.

EXAMPLE I.

What is the price of 65 bolls of Wheat, at 15 *s.* 10 *d.* 2 *f. per* boll?

65=8×8+1, that is the component parts of 64, the compoſite number leſs than 65, and neareſt to it are 8 and 8, and the difference betwixt 65 the *multiplier*, and the compoſite number 64, is 1; or the component parts of 65 are 8 and 8, and 1 over.

Here I multiply the ſum given by 8, and that *product* by 8, that new *product* gives 50 *l.* 16 *s.* the price of 64 bolls, to which I add 15 *s.* 10 *d.* 2 *f.* the price of the 1 boll over; ſo I have the price of 65 bolls, 51 *l.* 11 *s.* 10 *d.* 2 *f.*

L.	*s.*	*d.*	*f.*
0	15	10	2
			8
6	07	00	0
	8		
50	16	00	0
0	15	10	2
51	11	10	2

EXAMPLE II.

What is the price of 88 Hogſheads of Wine, at 17 *l.* 16 *s.* 7 *d.* 1 *f. per* Hhd? 88=9×9+7

L.	*s.*	*d.*	*f.*	
17	16	7	1	
			9	
160	09	5	1	
			9	
1444	04	11	1	= 81 hhds. or 9 times 9 hhds.
124	16	2	3	= the odd 7 hhds.
1569	01	2	0	= the whole 88 hhds.

L.	*s.*	*d.*	*f.*	
17	16	7	1	
			7	
124	16	2	3	= 7 hhds.

N.B. The choice of component parts is arbitrary; for here I might have made 88=8×5×2+8, where the next leſſer compoſite number is 80, and *difference* 8.

L.	*s.*	*d.*	*f.*		
17	16	7	1		
			8		
142	12	10	0	=	8 hhds.
		5			
713	04	02	0	=	40 hhds.
		2			
1426	8	4	0	=	80 hhds.
142	12	10	0	=	8 hhds.
1569	01	2	0	=	88 hhds.

Here to 1426 *l.* 8 *s.* 4 *d.* the price of 80 hogſheads, I add my firſt *product*, 142 *l.* 12 *s.* 10 *d.* the price of 8, and I have the *anſwer*,

1569 *l.* 1 *s.* 2 *d.*

If this method of multiplying the given price by the component parts of the *Integer*, is not, in every caſe, ſo expeditious as the Practice by the aliquot parts of a Pound; it is, at leaſt, in all caſes, as certain, and in many caſes, *viz.* where the price is not an aliquot part, leſs laborious, conſidering, that we can, at pleaſure, chuſe the component parts of the *Integer*.

Beſides, this method extends to all Mets and Meaſures whatſoever.

EXAMPLE.

What is the Weight of 48 cheſts of goods, each cheſt weighing 8 Pounds, 7 Ounces, 18 Penny-weight, 16 Grains Troy-weight.

Here 48=6×8.

24 *grs.* = 1 *pw.*
20 *pw.* = 1 *oz.*
12 *oz.* = 1 *lb.*

lb.	*oz.*	*pw.*	*gr.*
8	7	18	16
			6
51	11	12	0
			8
415	08	16	0

Here I ſay, 6 times 16 is 96, which is juſt 4 twenty fours, or 4 penny weights, and nothing over; ſo I ſet down 0, and carry 4 to the next place; then, 6 times 8 is 48, and 4 that I carried, make 52, I ſet down 2, and carry 5; then 6 times 1 is 6 Tens, and 5 that I carried, make 11 Tens, which make juſt 5 Twenties, or 5 Ounces, and 1 Ten over, which I ſet down, and carry the 5 Twenties, or 5 Ounces, to the place of Ounces, &c.

The component parts of any number below 100, or even above 100, are eaſily found: But this method is not confined to ſuch narrow bounds, but may eaſily be extended to higher numbers, if we conſider that

$$10=2\times5$$
$$50=2\times5\times5$$
$$100=4\times5\times5$$
$$1000=8\times5\times5\times5$$

Conſequently $10000=8\times5\times5\times5\times2\times5$

EXAMPLE.

What is the price of 1000 ſtones of Hay, at 4 *d.* 1 *f.* *per* ſtone? $1000=8\times5\times5\times5$

	L.	*s.*	*d.*	*f.*
	0	0	4	1
				8
	0	2	10	0
			5	
	0	14	2	0
			5	
	3	10	10	0
			5	
Anſwer - - - - - - - - -	17	14	2	0

DI-

DIVISION.

DIVISION diſcovers how often one number is contained in another number.

That number which is contained in the other, is called the *Diviſor*, that which contains it, the *Dividend*, and the number of times the *Dividend* contains the *Diviſor*, is called the *Quot*, or *Quotient.* We may then take the Definition of *Diviſion*, from its eſſential property, thus:

DIVISION, from a *Dividend* and *Diviſor* given, finds a third number, called the *Quot*, which contains unity as often as the *Dividend* contains the *Diviſor*.

The mark of *Diviſion* is ÷, *viz.* it ſets the *Dividend* above, and the *Diviſor* below a line, thus:

$$\begin{matrix}\textit{Dividend}\\ \textit{Diviſor}\end{matrix}\ \frac{12}{3}=4\,\textit{Quot.}$$

The common way of working *Diviſion*, is as in the margin, where I ſay, How often 3 in 12? *Anſ.* 4 times, this 4 I ſet in the *Quot*; then multiply the *Quot* and *Diviſor*, ſaying, 4 times 3 makes 12, which I ſet down below the *Dividend*, and ſubtract it from the *Dividend*, and 0 remains.

3)12(4
12
—
0
—

If I have 2968 to be divided by 8, as in the margin, I ſay, How often 8 in 29? (the left hand part of the *Dividend*, that contains the *Diviſor*) *Anſ.* 3 times; then I multiply, 3 times 8 is 24, which I ſet down below 29, and ſubtracts, there remains 5, to which I bring down the following figure 6, which makes 56; then I ſay, How often 8 in 56? *Anſ.* 7 times, I multiply, 7 times 8 is 56, which I ſet down below, and ſubſtracts, 0 remains; I bring down the next figure 8, and ſay, How often 8 in 8? *Anſ.* 1; I multiply and ſubtract, and 0 remains: So the *Quot* is 371.

```
8)2968(371
  24..
  ----
   56
   56
   ----
    08
     8
   ----
     0
   ----
```

N. B. The left hand part of the *Dividend*, *viz.* 29, which contains my *Diviſor* 8, I call my firſt *dividual*, and having brought the next figure in the *Dividend* down to the *Remainder*, that number 56, is my next *dividual*.

Hence, as many *dividuals* as are in the *Dividend*, ſo many figures are in the *Quot*, or the firſt dividual, with the number of figures ſubſequent to it in the *Dividend*, is equal to the number of figures or places in the *Quot*.

The reaſon of this operation is manifeſt; for the firſt *dividual* 29, as it ſtands in the *Dividend*, is 2900, and as 29 contains 8 thrice, ſo 2900 contains 8 300 times, and 500 remains: My next *dividual* 56, then is 560, or 56 Tens; and as 8 is contained in 56 7 times, ſo 8 in 560, or 56 Tens, 70 times, ſo the operation may ſtand thus.

```
8)2968(300
  2400  70
  ----   1
   568  ---
   560  371
  ----
     8
     8
  ----
     0
  ----
```

It is plain, the ſum of the 3 *Quots,viz.* 300+70+1=371, is the number of times the whole *Dividend* 2968 contains the *Diviſor* 8.

From the preceeding operation it appears, that the whole ſteps in *Diviſion* are contained in the following *monaſtich.*

1 2 3 4
Dic quot, *multiplica*, *ſubduc*, *tranſferq;* *ſequentem*.

Engliſhed thus,

First aſk, How often the *Diviſor*'s got?
The anſwer gives the figure in the *Quot*;
Subtract the *product* of theſe two, and then
Bring down the next, and aſk, How oft again?

Here I work as in the preceeding example, and finding 3 remain, I draw a line on the right hand of the *Quot*, I ſet the 3 of *Remainder* above the line, and the *Diviſor* 6 below, which forms a fraction $\frac{3}{6}$, to be expreſſed thus, 3 ſixth parts, or 3 parts, of which the whole contains 6. The number above the line is called the *numerator* of the fraction, and that below it the *denominator*.

$$
\begin{array}{l}
6)25797(4299\frac{3}{6} \\
\ \ 24\cdot\cdot \\
\hline
\ \ \ \ 17 \\
\ \ \ \ 12 \\
\hline
\ \ \ \ \ 59 \\
\ \ \ \ \ 54 \\
\hline
\ \ \ \ \ \ 57 \\
\ \ \ \ \ \ 54 \\
\hline
\ \ \ \ \ \ \ 3\ \textit{rem.} \\
\hline
\end{array}
$$

Every *diviſion* (as the preceeding example) which has its diviſor a digit, may be wrought thus; as I cannot get 6 in 2, I take in the following figure in the *Dividend* 5, and ſay, How often 6 in 25? *Anſ.* 4 times; then I ſet 4 below 5 in 25, and ſay, 4 times 6 is 24, from 25, and 1 remains, this 1 I ſuppoſe placed as 1 Ten on the left hand of the immediate following figure 7, which makes 17, then, how often 6 in 17? *Anſ.* 2, I ſet the 2 below the 7, and multiply thus, 2 times 6 is 12, from 17, and 5 remains, which I conceive as 5 Tens on the left hand of the following figure 9, makes 59; then, how often 6 in 59? *Anſ.* 9 times, &c.

$$
\begin{array}{l}
6)25797 \\
\ \ \ 4299\frac{3\ \text{numerator,}}{6\ \text{denominator.}}
\end{array}
$$

If, as in the margin, the *Diviſor* conſiſt of two places, I firſt conſider if my *Diviſor* is contain'd in the two left hand figures of the *Dividend* 87, and find it is; ſo I make the 2 anſwer to 8, and 6 to 7, and ſay, How often 2 in 8? *Anſ.* 3 times, and 2 remains, which I conceive as 2 Tens in the left hand of the 7, makes 27; then I ask, Will I get 6 three times in 27? Yes, conſequently I get 3 times 26 in 87, ſo I ſet 3 in the *Quot*, and multiply that 3 into the *Diviſor* 26, the *Product* 78 I ſet below 87, and ſubtract 78 from 87; to the *Remainder* 9 I bring down 6, the following figure in the *Dividend*, which makes 96; then I ſay, How ofren 2 in 6? *Anſ.* 4 times, and 1 remains, which, placed as 1 Ten on the left hand of 6, makes 16; then I ask, Will I likewiſe get 4 times 6, the following figure in the *Diviſor*, in 16? No; therefore I cannot get 4 times 26 in 96; I try 3 times; thus, 3 times 2 is 6, from 9, and 3 remains, which, placed as 3 Tens on the left hand of 6, makes 36, and 3 times 6 in 36, conſequently I find I will get 3 times 26 in 96, I ſet 3 in the *Quot*, I multiply that 3 into 26 the *Diviſor*, and ſet the *Product* 78 under

```
26)876568(33714 4/26
   78....
   --
    96
    78
    --
    185
    182
    ---
      36
      26
      --
      108
      104
      ---
        4
```

under 96, then ſubtract, and to the *Remainder* 18 I bring down 5, the following figure in the *Dividend*, makes 185, to be divided by 26, *viz.* a *Dividend* of three places by a *Diviſor* of two; therefore 2, the left hand figure of the *Diviſor*, muſt anſwer to 18, the two left hand figures of the *Dividend*; ſo I proceed thus, How oft 2 in 18? *Anſ.* 9 times, and 0 remains; but I cannot find 9 times 6 in 5, therefore I cannot find 9 times 26 in 185, I try it 8 times; thus, 8 times 2 is 16, from 18, and 2 remains, which on the left hand of 5, makes 25; but I cannot get 8 times 6 in 25, therefore I cannot get 8 times 26 in 185, I try it 7 times; thus, 7 times 2 is 14, from 18, and 4, which makes 45; but will I find 7 times 6 in 45? Yes, conſequently I have 7 times 26 in 185, ſo I ſet 7 in the *Quot*, and multiply and ſubtract as before; to the right hand of the *Remainder* I bring down the following figure in the *Dividend*, &c. Thus I proceed continually, till all the figures in my *Dividend* are brought down; ſo my diviſion is finiſhed. The laſt *Remainder* is the numerator of a fraction, and the *Diviſor* its denominator, as was ſhown in the preceeding example.

If your *Diviſor* conſiſts of three, or more places, work as in the preceeding examples, placing always the *Remainder* of each ſubtraction as ſo many Tens on the left hand of the following figure of the *Dividend*, if that *Remainder* is leſs than 9, but if it is 9, or more, ſet the figure you aſſumed in the *Quot*, without any further trial.

Example, If I am to divide 2898596 *l.* among 368 men,

Firſt, I find my *diviſor* of three places 368, is not contained in 289, the three leſt hand places of the *Dividend*, ſo I take in a 4th place 8, and enquire, How often 368 is contained in 2898, ſo 3, the leſt hand figure of my *Diviſor*, muſt anſwer to 28 the two leſt hand figures of my *Dividend*; then I proceed thus;

How often 3 in 28? *Anſ.* 9 times; I multiply, 9 times 3 is 27, from 28, and 1 remains, which 1, on the leſt hand of the following figure 9, makes 19; I cannot get 9 times 6 in 19, ſo I take 1 leſs, *viz.* 8 times, and ſay, 8 times 3 is 24, from 28, and 4 remains, which makes 49; but can I get 8 times 6 in 49? Yes, 8 times 6 is 48, from 49, and 1 remains, which, on the leſt hand of the fol-

```
368)2898596(7876
    2576...
    ----
     3225
     2944
     ----
      2819
      2576
      ----
       2436
       2208
       ----
        228 rem.
         20
       ----
368)4560(12 ſh.
    368.
    ----
     880
     736
     ---
     144 rem.
      12
     ---
     288
    144
    ----
368)1728(4 d.
    1472
    ----
     256 rem.
       4
    ----
368)1024(2 288/368
     736
     ---
     288
```

lowing figure 8 in the *Dividend*, makes 18; but will I get 8 times 8 in 18? No; ſo I take 1 leſs, *viz.* 7 times, and ſay, 7 times 3 make 21, from 28, and 7 remains, which makes 79; but will I get 7 times 6 in 79? Yes, 7 times 6 make 42, from 79, and more than 9 remains, ſo I infallibly find 7 times 368 in 2898, I ſet down 7 in the *Quot*, I multiply and ſubtract, and to the right hand of the *Remainder* I bring down 5, the following figure in the *Dividend*, ſo I have 3225, to be divided by 368; ſo I ſay, How often 3 in 32? *Anſ.* 9 times; then 9 times 3 make 27, from 32, and 5 remains, which makes 52; but I cannot find 9 times 6 in 52; ſo I take 8 times, and ſay, 8 times 3 is 24, from 32, and 8 remains, which makes 82; but will I find 8 times 6 in 82? Yes, and more than 9 remains: So I ſet 8 in the *Quot*, &c.

Inſtead of forming my fraction $\frac{228}{368}$, I reduce the 228 *l.* (which remain to be divided amongſt 368 Men) to Shillings, *viz.* I multiply 228 *l.* by 20, the number of Shillings in a Pound, and and divide the *Product* 4560 Shillings by 368, the *Quot* gives 12 Shillings to each man, and 144 Shillings to be divided amongſt them; theſe 144 Shillings multiplied by 12 give 1728 Pence, which, divided by 368, give 4 Pence to each man, and 256 Pence remain to be divided, which 256 Pence, multiplied by 4, give 1024 Farthings, which divided by 368, give to each man 2 Farthings, and $\frac{288}{368}$ of a Farthing; ſo each man has in all 7876 *l.* 12 *s.* 4 *d.* 2 *f.* $\frac{288}{368}$.

The ſame way, if I have 76548 Acres of ground, to be divided amongſt 245 men, I find each man has 312 Acres, and 108 remain, which, multiplied by 4, give 432 Roods, theſe divided by 245, give 1 Rood to each Man, and 187 Roods remain, which, multiplied by 40, give 7480 Falls, which, divided by 245, give to each Man 30 Falls, and $\frac{130}{245}$ of a Fall; ſo each Man has in all 312 *A.* 1 *R.* 30 $\frac{130}{245}$ *F.*

```
245)76548(312 A.
    735··
    ----
     304
     245
     ----
      598
      490
      ----
      108 remains.
        4
      ----
245)432(1 R.
    245
    ----
    187
     40
    ----
245)7480(30 130/245 F.
    735·
    ----
     130
     ----
```

If there are cyphers on the right hand of the *Diviſor*, (as in the margin) cut them off, and as many figures (68) from the right hand of the *Dividend*; divide (8479) what remains in the *Dividend* by (36) the ſig-

```
36|00)8479|68(235 1968/3600
      72···
      ---
      127
      108
      ---
       199
       180
       ---
       1968
       ----
```

nificant figures in the *Diviſor*, and to the right hand of the *Remainder* (19) annex the figures cut off from the *Dividend*, and you have 1968 for the numerator of your fraction, and the whole *Diviſor* 3600 for its denominator.

If there are cyphers cut off from both *Diviſor* and *Dividend*, the *remainder* of your *Diviſion* is the numerator of your fraction, and the ſignificant figures in your *Diviſor* its denominator, without regarding the cyphers cut off; the reaſon will appear afterwards in the doctrine of *Fractions*.

```
35|00)87960|00(2513 5/35
          70· ··
          ——
          179
          175
          ———
           46
           35
           ——
           110
           105
           ———
             5
            ——
```

Here I ſhall give one compend in *Diviſion*, *viz.*

To divide any number by a number conſiſting entirely of Nines, as 99, 999, &c.

Set the firſt figure of the *Quot* below the right hand figure of the firſt *Dividual*; add the *Quot* and right hand figure; the ſum, if leſs than 10, is the right hand figure of the *Remainder*; the other figures of the *Dividual* (except the left hand figure, which I cut off with a daſh of my pen) complete the *Remainder*: But if the ſum of the figure in the *Quot*, and the right hand figure of the *Dividual*, is 10, or more, take 0, or the exceſs above 10, for the right hand figure of the *Remainder*, and carry the 10 to the next figure in the *Dividual* for the next figure in the *Remainder*; only, if by thus carrying your 10, you exceed the *Diviſor*, the reſt of the Figures in the *Remainder* are 000, &c.

Example 1.	*Ex.* 2.	*Ex.* 3.	*Ex.* 4.
$\cancel{5}$ 75 *rem.*	92	04	069
99)$\cancel{3}$72	99)$\cancel{4}$88	99)$\cancel{5}$98	99)$\cancel{7}\cancel{7}$89
3 *Quot.*	4	6	8[illegible]

Ex. 5.	*Ex.* 6.
0	$\cancel{2}$0
$\cancel{7}$9 6	$\cancel{8}$1968 *remains* 68
99)$\cancel{1}\cancel{1}$65 8	999)2$\cancel{5}\cancel{7}$9486
14 8	2582 *Quot.*

In *Example* 6th, in my firſt *Dividual* 2579, I find 999 twice, I ſet the *Quot* 2 below 9, and add 2 and 9, make 11, I ſet 1 above, and carry my 10 to the next figure in the *Dividual*, *viz.* 1 and 7 make 8; I cancel the [redacted] figure 2, ſo my *Remainder* is 581, and my next *Dividual* 5814, in which 5814, I find 999 five times, I ſet 5 below; then I add 5, and 4 make 9, which I ſet above, and cancel 5, [redacted] figure of the *Dividual*; ſo the *Remainder* is 819, and my next *Dividual* 8198; &c.

Left hand

left hand

The reaſon is plain; for (in *Example* 1ſt) in dividing 372 by 100, the *Quot* is 3, and 72 remain; but 3×99 wants 3 of 3×100, theſe 3 then added to 72, make 75 the *Remainder*, when 372 is divided by 99.

$$
\begin{array}{rrrrrl}
 & L. & s. & d. & f. & L.\ \ s.\ \ d.\ \ f. \\
8) & 178 & 15 & 8 & 2 & (22\ \ 6\ \ 11\ \ 2\tfrac{2}{8} \\
 & 16\cdot & 40 & 84 & 16 & \\
\hline
 & 18 & 55 & 92 & 18 & \\
 & 16 & 48 & 8\cdot & 16 & \\
\hline
 & 2 & 7 & 12 & 2 & \\
 & 20 & 12 & 8 & & \\
\hline
 & 40s. & 84d. & 4 & & \\
 & & & 4 & & \\
\hline
 & & & 16f. & &
\end{array}
$$

In diviſion of *Integers* and their known parts by integers, firſt divide your integers, (as in the preceeding example) and reduce the *Remainder* 2 to the immediately following parts, *viz.* Shillings, add theſe to the 15 Shillings in the example, divide that ſum, and you have 6 Shillings in the *Quot*, which reduce to the next parts, *viz.* Pence, theſe add to the 8 Pence in the *Dividend*, that ſum divided by your *Diviſor* 8, give 11 Pence to the *Quot*, &c.

N. B. If the *Diviſor* is a digit, the *Quot* may be ſet below the *Dividend*; for any one may eaſily, in their Mind, reduce the remaining parts to the immediately following parts.

	L.	s.	d.	f.
8)	178	15	8	2
	22	6	11	$2\frac{2}{8}$ *Quot.*

L.	s.	d.	f.	L.	s.	d.	f.
72)9763	16	6	2	(135	12	2	$0\frac{26}{72}$
72..	860	144	24				
256	876	150	26				
216	72.	144					
403	156	6					
360	144	4					
43	12	24 *f.*					
20	12						
860 *s.*	24						
	12						
	144 *d.*						

It is plain from the preceeding *Example*, that the *Diviſor* is a compoſite number, there is no need of reducing it to its component parts; on the other hand, that *Reduction* is oftimes incovenient, becauſe the component parts do not always divide the loweſt denomination without a *Remainder*, and the fraction ariſing from that *Re-*

mainder is troubleſome, eſpecially to young Beginners.

Having thus finiſhed the Arithmetick of *Integers*, we ſhall apply it to uſe in the following Rules.

Of Geometric *Proportion.*

The ratio, or rate of two numbers, is the relation any one of theſe numbers bears to the other as to their magnitude, *i. e.* how often the one contains or is contained in the other; as 6 to 2, or the ratio of 6 to 2, is, that 6 contains 2 thrice.

That number which bears relation to the other, is called the *Antecedent*, and that to which it bears relation, is called the *Conſequent*; ſo, in the ratio of 6 to 2, the *Antecedent* is 6, and the *Conſequent* 2; the ſame way, in the ratio 3 to 12, the *Antecedent* is 3, and the *Conſequent* 12.

The meaſure of the ratio is the *Quot* ariſing from the diviſion of the *Antecedent* by the *Conſequent*: Thus, 3 is the meaſure of the ratio of 6 to 2, and $\frac{1}{4}$ the meaſure of the ratio of 3 to 12, which ſignifies the *Antecedent*, 3, to be one fourth of 12.

Hence ratios are written in the ſame manner as *Diviſion* is marked, *viz.* the *Antecedent* above, and the *Conſequent* below a line, thus $\frac{6}{2}$, ſignifies 6 to 2, or the ratio of 6 to 2, and $\frac{3}{12}$ the ratio of 3 to 12, or, ſimply, 3 to 12.

Two ratios are ſaid to be ſimilar, like, or equal one to the other, when their meaſures are the ſame; thus, the ratio of 6 to 2, is ſimilar to that of 24 to 8; for 3 is the meaſure of both ratios.

Similar, or equal ratios are written thus, $\frac{6}{2}=\frac{24}{8}$, *i.e.* the ratio of 6 to 2, is ſimilar or equal to the ratio of 24 to 8.

If two ratios are ſimilar, the four terms are ſaid to be proportional; ſo that

Proportion is the ſimilitude of ratios.

Proportionals are written thus, 6 : 2 :: 24 : 8, and read thus, 6 is to 2 as 24 to 8, or, as 6 is to 2, ſo is 24 to 8.

If any two terms are taken, as antecedent and conſequent of a ratio, theſe terms multiplied by any number produce a new ratio, ſimilar or equal to the firſt ratio: Let the terms be 3 and 5, where 5 contains 3 once, and two thirds of 3; multiply both by 4, and we have other two terms 12 and 20, where 20 contains 12 once, and 8 over, which 8 is two thirds of 12; ſo theſe terms are proportional, *viz.* 3 : 5 :: 12 : 20.

Definition. Alternate *proportion* takes the antecedent of one ratio as an antecedent ſtill to the antecedent of the other ratio, as a conſequent, and the conſequent of that firſt ratio, as an antecedent to the conſequent of the other ratio as a conſequent ſtill; thus, if 6 : 2 :: 24 : 8, then alternately 6 : 24 :: 2 : 8.

If 4 terms are proportional, they ſhall be alternately ſo; thus, if 3 : 5 :: 12 : 20, then 3 : 12 :: 5 : 20; for 12 contains 3 four times, and 20 contains 5 four times.

N. B. If the *product* of any two *factors* is divided by one of theſe 6 *factors*, the *Quot* is the other *factor*; thus, 24, the *product* of 4 and 6, being divided by 4, gives the other *factor* 6 in the *Quot*, and the ſame *product* 24, divided by one *factor* 6, gives the other *factor* 4 to the *Quot*.

If 4 terms are proportional, the *product* of the *extremes* is equal to the *product* of the *mdidle terms*; thus, if 6 : 2 :: 24 : 8, then 6×8=48=2×24.

N. B. The caſe is the ſame, if the terms are taken alternately, *viz.* if 6 : 24 :: 2 : 8; then 6×8=48=24×2: Hence, in *Proportionals*, we have this

GENERAL RULE.

Three terms given to find a fourth *Proportional*;

Multiply the two laſt terms, and divide the product *by the firſt, the* Quot *gives the fourth* Proportional.

EXAMPLE.

Given 8, 5 and 64, to find a fourth *proportional*, call the 4th *proportional* *x*.

$$\begin{array}{l} 8 : 5 :: 64 : x \\ \qquad\qquad\;\; 5 \\ \hline 8)320(40 = x \text{ the 4th } \textit{proportional.} \\ \quad 32 \\ \hline \quad 00 \end{array}$$

The reaſon is plain, for, calling the 4th *Proportional*, as it is unknown, *x*, the two middle terms are 5 and 64, conſequently their product 320, is the ſame with the product of 8 and x the extremes, which, divided by 8, one of the *factors*, gives 40$=x$, the other *factor*.

REDUCTION.

REDUCTION expreſſes the ſame value in different numbers under different names.

Thus, by *Reduction*, we find, that 3 Pounds are the ſame value with 60 Shillings.

Reduction is either *deſcending*, *aſcending*, or *mixt*.

Reduction deſcending, brings a greater name to a leſſer, when the greater contains the leſſer a juſt number of times, as Pounds to Shillings, where the Pound contains the Shilling juſt 20 times; or pounds weight to ounces, where the Pound contains the ounce juſt 16 times.

Reduction aſcending, brings a leſſer name to a greater, when the leſſer is contained in the greater a juſt number of times, as Shillings to Guineas, where the Shilling is contained in the Guinea juſt 21 times; or hours to days, where the hour is contained in the day juſt 24 times.

Mixt Reduction brings a greater name to a leſſer, or a leſſer to a greater, when the leſſer is not contained in the greater a juſt number of times, as Guineas to Pounds, where the Guinea contains the Pound once, and a Shilling more; or Merks

Scots to Pounds *Scots*, where the Merk is contained in the Pound juſt once, and a half more.

RULE I. To work *Reduction deſcending.* *Multiply by the value of the greater name.*

RULE II. To work *Reduction aſcending.* *Divide by the value of the greater name.*

RULE III. To work *mixt Reduction.* *Find a third name, that is contained both in the name given, and in the name ſought, a juſt number of times; reduce the name given to that third name by* Rule I. *then reduce that third name to the name ſought, by* Rule II.

EXAMPLE. I.

To reduce 796 *l.* to Shillings, Pence and Farthings.

Here the Queſtion is *deſcending*; for the Pound contains the Shilling 20 times, the Shilling contains the Pence 12 times, and the Penny contains the Farthings 4 times; therefore I multiply my 796 *l.* by 20, which gives 15920 Shillings, they multiplied by 12 give 191040 Pence, which, multiplied by 4, give 764160 Farthings.

796 *l.*
20
15920 *s.*
12
3184
1592
191040 *d.*
4
764160 *f.*

EXAMPLE II.

To reduce 798 *l.* 15 *s.* 8 *d.* to *Farthings*.

Here my 798 *l.* give 15960 Shillings, to which I add the 15 odd Shillings in the Queſtion, which make 15975 *s.* theſe reduced to Pence, give 191700 *d.* to theſe I add the odd 8 Pence in the queſtion, which make 191708 *d.* which, reduced to Farthings, give 766832 *f.*=798 *l.* 15 *s.* 8 *d.*

```
   798 l.
    20
  ------
  15960
    +15 s.
  ------
  15975 s.
     12
  ------
  31950
 15975
 -------
 191700
     +8
 -------
 191708 d.
      4
 -------
 766832 f.
```

EXAMPLE III.

To reduce 7648428 Farthings to Pounds, I reduce firſt my Farthings to Pence, my Pence to Shillings, and then my Shillings to Pounds.

The queſtion is *Reduction aſcending*; for I muſt reduce Farthings, a leſſer name, to Pence a greater, where the Penny contains the Farthing juſt 4 times, &c.

```
 4)7648428 farth.
12)1912107. 3 d. rem.
2|0) 15934|2 s.
      7967 l.
```

E X-

EXAMPLE IV.

To reduce 8796 Guineas to Pence, and back again.

Here, by *Reduction deſcending*, I reduce my 8796 Guineas, firſt to Shillings, then to Pence, and find 2216592 Pence, which I, by *Reduction aſcending*, reduce firſt to Shillings, then to Guineas; ſo theſe two *Rules* are Proofs of each other.

```
         8796 Guineas.
           21
         ----
         8796
        17592
       ------
       184716 s.
           12
       ------
       369432
      184716
      -------
  12)2216592 d.
  21)184716 s. (8796 gu.
     168
     ---
      167
      147
      ---
       201
       189
       ---
        126
```

EXAMPLE V.

To reduce 25796 Merks *Scots* to Pounds *Scots*.

N. B. Three half Merks are equal to one Pound *Scots*; ſo I reduce my Merks to half Merks, and, by the *Rule aſcending*, I reduce my half Merks to Pounds, and 1 remains, which is $\frac{1}{3}$ of a Pound, or 1 half Merk=6 *s.* 8 *d. Scots*.

```
   25796 merks.
       2
   -----
 3)51592 half merks.
   17197⅓ pounds.
```

It is plain this *Example* is of the *mixt Rule*, the greater name, *viz.* the Pound, not containing the Merk a juſt number of times; the half Merk is the third name which is contained in the name given, *viz.* the Merk, twice, and in the name ſought, *viz.* the Pound, 3 times juſt.

EXAMPLE VI.

In 9768 *l Scots*, how many *Scots* Merks?

The queſtion is *mixt*, the third name, the half Merk, I reduce my Pounds to half Merks, multiplying by 3, and theſe half Merks to Merks, dividing by 2.

9768 *l. Scots.*
3
———
2)29304 *half merks.*
14652 *merks.*

N. B. The method is the ſame in reducing Pounds and Merks *Sterling*; for the Merk *Sterling* is 13 *s.* 4 *d.*

EXAMPLE VII.

To reduce 768 *l. Sterling* to Merks *Scots.*

The queſtion is *deſcending.*

The Pound *Sterling* contains juſt 18 Merks *Scots.*

768 *l.*
18
———
6144
768
———
13824 *merks Scots.*

EXAMPLE VIII.

In 75648 Merks *Scots*, how many Pounds *Sterling*? *Anſwer*, 4202 *l.* 13 *s.* 4 *d.*

The queſtion is *aſcending*; ſo I divide by 18 Merks in the Pound, the *Quot* gives 4202 *l.* and 12 Merks remain, which, multiplied by 20 Shillings, and theſe divided by 18, give 13 Shillings, and 6 remain; theſe multiplied by 12 Pence, give 72, which, divided by 8, give 4 Pence.

```
18)75648(4202 l. Scots. Sterling
   72...
   --
    36
    36
    ---
     048
     36
     --
      12
      20
      --
18)240(13 s.
   18.
   --
    60
    54
    --
     6
    12
    --
18)72(4
   72
   --
    0
```

EXAMPLE IX.

In 9768000 pieces of Eight, how many Pounds *Sterling*, at 4 *s.* 6 *d.* *per* Piece?

The queſtion is *mixt*; the 3d name is 6 Pence, for there are juſt 9 Sixpences in the piece of eight, and 40 in the Pound.

```
    9768000
          9
   --------
4|0)8791200|0
    2197800 l. ſterl.
```

EXAMPLE X.

In *796 l. Scots*, how many *French* Livres, at 15$\frac{1}{2}$ Pence *per* Livre?

The queſtion is *mixt*; the 3d name is half Pence, for 40 half Pence make a Pound *Scots*, and 31 half Pence make a Livre.

Having reduc'd my *796 l. Scots* to 1027 Livres, there remains 3, which I multiply by 20, the number of Sols in a Livre, and divide the *product* 60 by 31, the *Quot* gives 1 Sol, and 29 remains, which multiplied by 12 Deniers in a Sol, and divided by 31, gives 11 Deniers, and 7 remains; ſo *796 l. Scots*=1027 Livres, 1 Sol, 11$\frac{7}{31}$ Deniers.

796 l.
40
31)31840(1027 *Livres.*
31···
084
62
220
217
3 *remains.*
20
31)60(1 *Sol.*
31
29 *remains.*
12
58
29
31)348(11 $\frac{7}{31}$ *den.*
31·
38
31
7 *remains.*

The preceeding *Examples* being well underſtood, I ſcarce think any caſe can happen in *Reduction*, but may be eaſy to the judicious reader.

N.B. The whole of *Reduction* may be brought to the General Rule of *Proportion*; thus,

EXAMPLE I.

I demand, How many Shillings and Pence are in 978 Pound?

The *Proportion* ſtands thus; If 1 *l.* give 20 *s.* what will 978 *l.* give?

1 *l.* 20 *s.* :: 978 *l.*
20
———
19560 *s.* *Anſw.*
———

Here the 1ſt term 1 neither multiplies nor divides.

Again, If 1 Shilling gives 12 Pence, what will 19560 *s.* give?

1 *s.* 12 *d.* :: 19560 *s.*
12
———
3912
1956
———
234720 *d.* *Anſw.*
———

Again, In 234720 Pence, how many Shillings and Pounds?

As 12 Pence is to 1 Shilling, ſo is 234720 *d.* to a 4th *Proportional.*

Here I only divide my Pence by 12; for multiplying by 1, produces the ſame number.

12 *d.* 1 *s.* :: 234720
1
12)234720
19560 *s.*

EXAMPLE II.

In 7642 Merks, how many Pounds?

Here the Pound, containing 3 half Merks, is to the Merk as 3 is to 2.

3 : 2 :: 7642
2
3)15284
$5094\frac{2}{3}$ Pounds.

GOLDEN RULE.

THE GOLDEN RULE is either *ſimple* or *compound.*

The *Simple Rule*, from three numbers given, finds a fourth *Proportional*; and is therefore called, *The Rule of Three:* It is either *Direct* or *Inverſe.*

N. B. In every ſimple queſtion there are three numbers, and two different ſpecies, or kinds of things.

EX-

EXAMPLE I.

If 5 *l.* is the annual intereſt of 100 *l.* what is the annual intereſt of 798 *l.*?

Here the three given numbers are 5, 100 and 798, the two different ſpecies, are *Principal* and *Intereſt*, and the ſpecies demanded in the queſtion is *Intereſt.*

EXAMPLE. II.

If 76 men ſpend 340 *l.* in any time, what will 15 men ſpend in that time, and at that rate?

Here the three numbers are obvious; the two ſpecies are *men* and *money*, and the ſpecies ſought is *money.*

EXAMPLE III.

If 36 yards be a rood of maſon-work, at 4 quarters high, what will the rood be at 7 quarters high?

The two ſpecies are *length* and *height*, and *length* is ſought.

EXAMPLE IV.

If 36 men do a piece of work in 40 days, in what time will 15 men do that work?

The ſpecies are *men* and *time*, and *time* is demanded.

RULE I. To ſtate the queſtion, *i. e.* to place the numbers in the *ſimple Rule.*

> *Set that number in the queſtion which is of the ſame ſpecies with the number ſought, in the ſecond place; that which gives it, in the firſt, and the third ſets itſelf.*

N. B. It is plain from the preceeding Rule, and the ſtating of the foregoing *Examples*, that, in the *Rule of Three*, the firſt and third terms are of the ſame ſpecies, and the ſecond and fourth, or term ſought, are of the ſame ſpecies.

Follows the ſtating of the four preceeding Examples, *according to the antecedent Rule.*

EXAMPLE I.

Pr. *Int.* *Pr.* *Int.*
100 : 5 :: 798 :

Here, becauſe *Intereſt* is demanded, I ſet my 5 *l.* of *Intereſt* in the ſecond place, my 100 *l.* of *Principal*, which gives that 5 *l.* of *Intereſt*, in the firſt, and the 798 *l.* of *Principal*, whoſe intereſt is ſought, ſets itſelf in the third place.

Here obſerve, once for all, that the firſt and third terms, 100 *Principal* and 798 *Principal*, are of the ſame ſpecies, or kind; and the ſecond and fourth terms, *viz.* 5 *Intereſt* and the *Intereſt* ſought, are likeways of the ſame ſpecies.

EXAMPLE II.

Men. *L.* *M.* *L.*
76 : 340 :: 15

Here *Money* being demanded, I ſet 340 *l.* which is of the ſame ſpecies with the number ſought, in the ſecond place, and 76 *Men*, which gives it, in the firſt.

EXAMPLE III.

Qurs. h. *Ells l.* *Qurs. h.* *Ells l.*
4 : 36 :: 7

Here *Length* being demanded, I ſet 36 Ells of Length in the ſecond place, and 4 Quarters of Height, which gives it, in the firſt.

EXAMPLE IV.

Men. *Days.* *Men.* *Days.*
36 : 40 :: 15

Here *Time* being demanded, I ſet 40 Days, which is the ſame ſpecies, in the ſecond place, and the 36 men, which do the work in that time, in the firſt.

RULE II. To know if a queſtion is of the *Direct* or *Inverſe* Rule.

> *If more give more, or leſs give leſs; the queſtion is* Direct.
>
> *If more give leſs, or leſs give more; the queſtion is* Inverſe.

In *Example* I. 798 *l.* being more *principal* than 100 *l.* muſt give more *intereſt*; ſo, as I go directly on from more *principal* to more *intereſt*, the queſtion is *Direct*.

In *Example* II. 15 *men* being a leſs number than 76 *men*, muſt ſpend leſs *money* in the ſame time, and at the ſame rate; therefore, as I go directly on from a leſs number of men to a leſs number of Pounds, the queſtion is *Direct*.

In *Example* III. 7 *Quarters* of *Height* being more than 4 *Quarters*, requires leſs Length to make the *Rood*; ſo, turning back from more Height to leſs Length, the queſtion is *Inverſe*, or backward.

In *Example* IV. 15 being a leſs number of *men* than 36, requires more time to do the ſame work, ſo leſs requiring more, the queſtion is *Inverſe*.

RULE III. To work the *Direct* Rule.

Multiply the two laſt numbers together, and divide the Product *by the firſt, the* Quot *gives the anſwer.*

RULE IV. To work the *Inverſe* Rule.

Multiply the two firſt numbers together, and divide the Product *by the laſt.*

The preceeding *Examples* are wrought thus;

Here I multiply 798 by 5, and divide the *Product* by 100, the *Quot* is 39 *l.* and 90 remains, which I multiply by 20 *s.* in a Pound, and divide the *Product* by 100, the *Quot* is 18 Shillings; ſo the *Intereſt* of 798 *l.* at 5 *per Cent.* is 39 *l.* 18 *s.* 0 *d.* 0 *f.*

Pr. *Int.* *Pr.* *Int.*
100 : 5 :: 798
5
———
1|00)39|90(39 *l.*
20
———
1|00)18|00(18 *s.*

Here I multiply the *Remainder* 8 by 20, and divide the *Product* 160 by 76, the *Quot* gives 2 Shillings, and 8 remains, which I multiply by 12 Pence in a Shilling, and divide the *Product* by 76, the *Quot* gives 1 Penny, and 20 remains, which I multiply by 4 Farthings in a Penny, and divide the *Product* 80 by 76, the *Quot* gives one Farthing and 4 remains; ſo the anſwer is 67 *l.* 2 *s.* 1 *d.* 1 $\frac{4}{76}$ *f.*

M. *L.* *M.* *L.*
76 : 340 :: 15
15
170
34
76)5100(67 *l.*
456
540
532
8 *rem.*
20
76)160(2 *ſh.*
152
8
12
76)96(1 *d.*
76
20
4
76)80(1 *f.*
76
4

Here the queſtion being *Inverſe*, I multiply the two firſt terms, *viz.* 36 by 4, and divide by 7. The *Remainder* 4, I multiply by 4 *Quarters* in an *Ell*, and divide by 7, the *Quot* gives 2 *Quarters* and $\frac{2}{7}$ of a *Quarter*; ſo the Anſwer is, 20 *Ells*, 2 *Quarters* and $\frac{2}{7}$.

q.h. *E.len.* *q.h.* *E.len.*
4 : 36 :: 7
4
—
7)144(20 *Ells.*
14·
—
04
4
—
7)16(2 *Quarters*
14
—
2

M. *D.* *M.* *D.*
36 : 40 :: 15
4
15)1440(96 *Days*, *Anſwer.*
135·
—
90
90
—
0
—

N. B. If the firſt or third term, or both, are of different denominations, reduce both to the loweſt denomination, continue the ſecond term (if it is of one ſimple denomination) the ſame, and you have the anſwer of the ſame denomination with the ſecond term.

EXAMPLE I.

If $4\frac{3}{4}$ Yards of Cloth coſt 5 *l.* what will 86 Yards coſt?

Having reduced my $4\frac{3}{4}$ Yards to 19 Quarters, and my 86 Yards to 344 Quarters, the Queſtion runs thus.

4 *Yards.*	86 *Yards.*
4	4
16	344 *qrs.*
3 *qrs.*	
19 *qrs.*	

If 19 Quarters of Cloth coſt 5 *l.* what will 344 Quarters coſt?

The ſecond term 5 *l.* being of one ſimple denomination, I continue the ſame, and the anſwer comes out Pounds.

Anſwer,
90 *l.* 10 *s.* 6 *d.* 1 $\frac{5}{19}$

```
 Qrs.     L.     Qrs.     L.
  19   :  5  ::  344
                   5
                ------
             19)1720 l. (90
                171·
                ----
                  10
                  20
                ----
             19)200(10 s.
                19·
                ---
                 10
                 12
                ---
             19)120(6 d.
                114
                ---
                  6
                  4
                ---
             19)24(1 f.
                19
                --
                 5
```

N. B. 2. If the ſecond term is of different denominations, reduce it to the loweſt denomination, and the anſwer comes out the ſame loweſt denomination.

E X-

EXAMPLE II.

What is the price of 75 bolls of Meal, at 9 *s.* 4 *d. per* boll.

I reduce my 9 *s.* 4 *d.* to 112 *d.* and ſay, If 1 boll give 112 *d.* what will 75 bolls give?

Anſ. 8400 *d.* which, reduced to Shillings and Pounds, gives juſt 35 *l.*

```
  9 s.
 12
 ——
108 d.
  4
 ——
112 d.
```

```
B. : d.  :: B.
1  : 112 :: 75
         75
        ———
        560
       784
       ————
    1) 8400(8400 d.
       ————
   12)8400(700 s.
      84
      ——
      000
      ————
   2|0)70|0 s.(35 l.
```

N. B. 3. The Rule called **Fellowship**, is nothing elſe but as many operations in the *Rule of Three*, as there are partners in trade. The firſt term is always the whole *Stock*, the ſecond the whole *Gain* or *Loſs*, and the third any one of the partner's ſtocks, in order to find his ſhare of the loſs or gain.

EXAMPLE III.

Three partners *A*, *B* and *C* give in their ſtocks to trade; *A* gives 536 *l.* *B* gives 320 *l.* and *C* 144 *l.* they gain 288 *l.* What is each man's ſhare of that Gain?

A's ſtock=536 *l.*
B's ——=320
C's ——=144

Whole ſtock=1000

Whole ſtock. *Gain.* A'*s ſtock.*
1000 : 288 :: 536
536
1728
864
1440
1|000)154|368(154 *l.*
20
1|000)7|360(7 *s.*
12
72
36
1|000)4|320(4 *d.*
4
1|000)1|280(1 $\frac{28}{100}$ *f.*

Whole ſtock. *Gain.* B'*s ſtock.*
1000 : 288 :: 320
320
576
864
1|000)92|160(92 *l.*
20
1|000)3|200(3 *s.*
12
1|000)2|400(2 *d.*
4
1|000)1|600(1 $\frac{60}{100}$ *f.*

Whole ſtock. *Gain.* *C's. ſtock.*
1000 : 288 :: 144
144

1152
1152
288

1|000)41|472(41 *l.*
20

1|000)9|440(9 *s.*
12

88
44

1|000)5|280(5 *d.*
4

1|000)1|120($1\frac{12}{100}$ *f.*

	L.	*s.*	*d.*	*f.*
A's *Gain*	154	7	4	$1\frac{28}{100}$
B's ——	92	3	2	$1\frac{60}{100}$
C's ——	41	9	5	$1\frac{12}{100}$
	288	0	0	0 00

Here I ſay, If 1000 *l.* ſtock give 288 *l.* gain, what will *A*'s ſtock, 536 *l.* give? *Anſ.* 154 *l.* 7 *s.* 4 *d.* $1\frac{28}{100}$ *f.* which is *A*'s ſhare of the gain;

after the ſame manner I find *B*'s ſhare and *C*'s ſhare, and for proof, I add theſe three ſhares together, and find they make up the whole gain 288 *l.* preciſely.

The Queſtions in the *ſimple Golden Rule* may be made all *Direct*, if ſtated thus.

Set that number which is of the ſame ſpecies with the number ſought, in the third place; and if more require more, ſet the leſſer of the other two given numbers in the firſt place, the greater in the ſecond.

If leſs gives leſs, ſet the greater of the other two given numbers in the firſt place.

If more gives leſs, ſet the greater of the other two given numbers in the firſt place.

But if leſs gives more, ſet the leſſer of the other two given numbers in the firſt place.

EXAMPLE I.

What is the Intereſt of 642 *l.* at 5 *per Cent. per Annum?*

Here I ſet 5 of Intereſt in the third place; and becauſe 642 of Principal requires more Intereſt than 100, I ſet my leſſer number 100 in the firſt place, and 642 in the ſecond. This proportion taken alternately, will ſtand as before, 100 *Pr.* : 5 *Int.* :: 642 *Pr.*

Pr. *Pr.* *Int.* *Int.*
100 : 642 :: 5
5
——
1|00)32|10(32 $\frac{10}{100}$

E X-

EXAMPLE II.

If 78 Men ſpend 250 *l.* in any time, what will 16 Men ſpend in that time, and at that rate?

Here, as below, becauſe a leſſer number of Men, *viz.* 16, require leſs Money, I ſet my greater number, 78, in the firſt place, and 16 in the ſecond. This proportion taken alternately, will ſtand thus, 78 *m.* : 250 *l.* :: 16 : *l.* conform to the *Rule* formerly laid down.

M. *M.* *L.* *L.*
78 : 16 :: 250
16
150
25
78)4000(51 $\frac{22}{78}$ *Anſwer.*
390
100
78
22

EXAMPLE III.

If 36 Ells are a Rood of work, at 4 Quarters high, what will the Rood be at 7 Quarters high?

Here 7 Quarters high requiring leſs Length to make a Rood than 4 Quarters high, I ſet 7, my greater Height, in the firſt place, and 4 in the ſecond; ſo the Queſtion becomes *Direct.*

Q.h. *Q.h.* *E. l.* *E. l.*
7 : 4 :: 36
4
7(144
20 $\frac{4}{7}$ *Anſ.*

EXAMPLE IV.

If 72 Men do a piece of work in 12 Days, in what time will 8 Men do that work?

Here 8 Men requiring more time to do the work than 72 Men, I ſet my leſſer number, 8, in the firſt place, and the queſtion becomes *Direct*.

```
M.   M.    D.
8 : 72 :: 12
          72
          --
          24
          84
          --
        8)864
          108 Anſw.
```

Thus the whole *Rule of Three* is reduced to the general Rule of *Proportion*, formerly laid down.

Here we ſhall illuſtrate the *multiplication* and *diviſion* of *Integers*, and their known parts by *Integers*, in an *Example* of the *Rule of Three*.

What is the intereſt of 78 *l.* 16 *s.* 6 *d.* at 5 *per Cent.*?

```
Pr.Int.Pr.
100:5::78 l.      16 s.      6 d.
                             5
   ----------------------------
100)3|94          02         6(3 l. 18 s. 9 d. 3 6/10 f.
       20       1880       984
     ----       ----       ---
     1880       18|82      9|90
                   12        4
                 ----      ---
                  164      3|60
                   82
                 ----
                  984
```

Examples of the ſimple Rule of Three.

Queſtion 1. If a man gains 2 Shillings in three days, what will he gain in twelve days at that rate? *Anſwer*, 8 Shillings.

Queſt. 2. If 3 ounces of ſilver be worth 15 Shillings, what is the price of 4 *lb.* of ſilver at that rate? *Anſw.* 240 Shillings, or 12 *l.* *Nota*, 12 ounces make a pound weight of ſilver.

Queſt. 3. If the intereſt of 100 *l.* for a year be 6 *l.* what is the intereſt of 530 *l.* for the ſame time? *Anſw.* 31 *l.* 16 *s.*

Queſt. 4. If 4 *lb.* of goods coſt 8 Shillings, what will be the price of 1 *Ct.* (112 *lb.*) at that rate? *Anſw.* 224 Shillings, or 11 *l.* 4 *s.*

Queſt. 5. If 15 *lb.* of cochenial coſt 24 *l. ſterl.* what will be the price of 82 *lb.* at that rate? *Anſw.* $131\frac{3}{15}$ *l.* = 131 *l.* 4 *s.*

Queſt. 6. What is the price of 1 piece of cloth, when 40 pieces coſt 750 *l.* 15 *s.*? *Anſw.* 18 *l.* 15 *s.* $4\frac{1}{2}$.

Queſt. 7. If 8 *lb.* of tobacco coſt 14 *s.* what will be the price of $\frac{1}{2}$ *Ct.* (56 *lb.*)? *Anſ.* 4 *l.* 18 *s.*

Queſt. 8. If I buy cloth for 16 *s.* and ſell it for 18 *s. per* yard, how much do I gain at that rate upon every 100 *l.*? *Anſw.* 12 *l.* 10 *s.*

Queſt. 9. If a dozen of ells of linen are valued at 3 *l.* 6 *s.* how much will 8 pieces (each piece containing 54 ells) amount to at the ſame rate? *Anſ.* 118 *l.* 16 *s.*

Queſt. 10. If I buy 2 *Ct.* 1 *qr.* 7 *lb.* of coffee for 64 *l.* 15 *s.* at how much muſt I ſell it out *per* ounce to gain 21 *l.* 11 *s.* 8 *d.* upon the whole? *Anſw.* 5 *d. per* ounce.

Queſt. 11. If $\frac{3}{4}$ of a yard of velvet coſt 7 *s.* 3 *d.* how many yards will be got for 13 *l.* 15 *s.* 6 *d.* *Anſw.* 28 $\frac{1}{2}$ yards.

Queſt. 12. What principal ſum will yield 35 *l.* 15 *s.* intereſt at 5 *l. per Cent.*? *Anſw.* 715 *l.*

Queſt. 13. In what time will 100 *l.* principal yield 73 *l.* intereſt, at 5 *l. per Cent. per annum*? *Anſw.* 14 years, 7 $\frac{1}{5}$ months.

Queſt. 14. If 150 pints of wine ſerve 15 men for ſix months, how much will 137 men drink in that time, at the ſame rate of drinking? *Anſw.* 1370 pints.

Queſt. 15. If 150 pints of wine ſerve 15 men for ſix months, how many men will drink 1370 pints in the ſame time, at the ſame rate of drinking? *Anſw.* 137 men.

Queſt. 16. If a man performs a journey in 9 days, when the day is 11 hours long, in what time will he perform the ſame, when the day is 15 hours long? *Anſw.* In ſix days 9 hours.

Queſt. 17. If the penny-loaf ought to weigh 9 ounces, when wheat is at 4 *s.* 6 *d. per* buſhel, what ought it to weigh when wheat is at 6 *s.* 9 *d. per* buſhel? *Anſw.* 6 ounces.

Queſt. 18. At what price is the buſhel of wheat, when the penny-loaf weighs 5 ounces 8 penny-weight, if it weighs 9 ounces, when wheat is at 4 *s.* 6 *d.*? *Anſw.* 7 *s.* 6 *d.* *N. B.* 20 penny-weight make 1 ounce.

Queſt. 19 If 42 *s.* worth of wine ſerve 12 men, when wine is ſold at 25 *l.* 4 *s.* *per* hogſhead, how many men will 42 *s.* worth ſerve when the wine is ſold at 18 *l.* 18 *s.* *per* hhd.? *Anſ.* 16 men.

Queſt. 20. A governor of a fort having proviſions ſufficient to ſerve 2820 men for ſix months, how many of them muſt he diſmiſs that the proviſions may ſerve for two months longer? *Anſw.* 705 men.

Queſt. 21. If I lend a man 650 *l.* for 22 months, how long ought he to lend me 953 *l.* 6 *s.* 8 *d.* to be even with me? *Anſw.* 15 months.

Queſt. 22. If 54 men build a wall in 94 days, what time would 36 men take to build the ſame wall, at the ſame rate of working? *Anſw.* 141 days.

Queſt. 23. If 14 *l.* worth of oats ſerve 57 horſes, when oats are at 9 *s.* *per* boll, how many horſes will eat to the ſame value when oats are at 11 *s.* 2½ *d.* *per* boll? *Anſw.* 45 horſes.

Queſt. 24. If 14 *l.* worth of oats ſerve 56 horſes, when oats are at 9 *s.* *per* boll, what will be the price of the boll of oats, when 45 horſes eat 14 *l.* worth at the ſame rate of eating? *Anſ.* 11 *s.* 2½ *d.*

Queſt. 25. If 55 men can build a wall in 16 days, when they work 10 hours every day, how many hours muſt they work every day, that 40 men may build the wall in the ſame time? *Anſw.* 13¾.

The

The GOLDEN RULE *compound*

From five numbers given finds a ſixth, and is therefore called, *The Rule of five.* It is either *Direct*, *Inverſe* or *Mixt*, and the *Mixt Rule* is of two ſorts, *viz.* 1. *Direct* and *Inverſe*, and, 2. *Inverſe* and *Direct.*

N. B. 1. Every *compound Queſtion* may be reduced into two *ſimple ones.*

N. B. 2. In every *compound Queſtion* there are five numbers given, and three different ſpecies.

EXAMPLE. I.

If 5 *l.* is the intereſt of 100 *l.* for 365 days, what is the intereſt of 798 *l.* for 86 days?

Here the three different ſpecies, or kinds of of things, are *Principal*, *Intereſt* and *Time*, and *Intereſt* is ſought.

EXAMPLE II.

If 12 inches of Length, and 12 of Breadth, require 12 of Thickneſs, to make a ſolid foot, what will 6 of Length, and 4 of Breadth require?

Here the three different ſpecies are *Length*, *Breadth* and *Thickneſs*, and *Thickneſs* is demanded?

EXAMPLE III.

If 12 men work 96 Roods of work in 15 days, in what time will 8 men work 120 Roods, at that rate?

Here the three ſpecies are *Men*, *Roods* and *Time*, and *Time* is demanded.

RULE I. To ſtate the queſtion in the *compound Rule.*

Set that number, which is of the ſame ſpecies with the number ſought, in the third place, that which gives it, in the firſt, the number common to both, in the ſecond, and that which is of the ſame ſpecies with the firſt number, in the fourth place; the fifth number ſets itſelf.

N. B. If the third term is duly ſet, it is no matter which of the other two ſpecies takes the 1ſt, which the 2d place, provided always the 1ſt and 4th terms be of the ſame ſpecies or kind.

Here, as below, becauſe *Intereſt* is demanded, I ſet 5 *l.* of *Intereſt* in the third place, the 100 *l.* of *Principal*, which gives that *Intereſt*, in the firſt, and the 365 days, which is common to both (for 100 *l. Principal* gives 5 *l. Intereſt* in that time, *viz.* 365 days) in the ſecond place; ſo my three different ſpecies are duly placed: Then I begin again with my firſt ſpecies, and ſet it in the fourth place.

In the ſecond ſtating, I ſet 365 days in the firſt, as giving 5 *l.* of *Intereſt*, and my 100 *l. Principal* in the ſecond place, as being common to both; for 365 days afford 5 *l. Intereſt* for 100 *l. Principal.*

EXAMPLE I.

Pr. *D.* *Int.* *Pr.* *D.* *Int.*
100 : 365 : 5 :: 798 : 86

Or thus,

D. *Pr.* *Int.* *D.* *Pr.* *Int.*
365 : 100 : 5 :: 86 : 798

EX-

EXAMPLE II.

In len. *In br.* *In th.* *In len.* *In br.* *Th.*
12 : 12 : 12 :: 6 : 4

Or thus,

In br. *In len.* *In th.* *In br.* *In len.* *Th.*
12 : 12 12 :: 4 : 6

EXAMPLE III.

R. *M.* *D.* *R.* *M.* *D.*
96 : 12 : 15 :: 120 : 8

Or thus,

M. *R.* *D.* *M.* *R.* *D.*
12 : 96 15 :: 8 : 120

RULE II. To reſolve any *compound Queſtion* into its two *ſimple ones.*

The firſt, third and fourth terms make the firſt ſimple Queſtion.

The ſecond, third and fifth terms make the ſecond ſimple Queſtion.

RULE III. To know if a Queſtion is of the *Direct*, *Inverſe*, or *Mixt Rule*; and if *mixt*, to know whether it is *Direct* and *Inverſe*, or *Inverſe* and *Direct*.

If both ſimple Queſtions *are* Direct, *the* compound Queſtion *is* Direct.

If both are Inverſe, *the* compound Queſtion *is* Inverſe.

If one ſimple Queſtion *is* Direct, *and the other* Inverſe, *the* compound Queſtion *is* Mixt.

If the firſt ſimple Queſtion *is* Direct, *and the ſecond* Inverſe, *the* compound mixt Queſtion *is* Direct *and* Inverſe, & *e contra.*

EXAMPLE I.

Pr. D. Int. Pr. D. Int.
100 : 365 : 5 :: 798 : 86
Pr. Int. Pr. Int.
1ſt Simp. Queſt. 100 : 5 :: 798
D. Int. D. Int.
2d Simp. Queſt. 365 : 5 :: 86

Here both *ſimple Queſtions* are *Direct*; the *Compound* is *Direct*.

EXAMPLE II.

In len. Br. Th. L. Br. Th.
12 : 12 : 12 :: 6 : 4
In len. th. len. th.
1ſt Simp. Queſt. 12 : 12 :: 6
br. th. br. th.
2d Simp. Queſt. 12 : 12 :: 4

Here both *ſimple Queſtions* are *Inverſe*; the *Compound* is *Inverſe*.

EXAMPLE III.

R. *M.* *D.* *R.* *M.* *D.*
96 : 12 : 15 :: 120 : 8
r. *d.* *r.* *d.*
1ſt Simp. Queſt. 96 : 15 :: 120
m. *d.* *m.* *d.*
2d Simp. Queſt. 12 : 15 :: 8

Here the firſt *ſimple Queſtion* is *Direct*, the ſecond *Inverſe*; the *Compound* is *Mixt*, or *Direct* and *Inverſe*.

Or thus,

M. *R.* *D.* *M.* *R.* *D.*
12 : 96 : 15 :: 8 : 120
m. *d.* *m.* *d.*
1ſt Simp. Queſt. 12 : 15 :: 8
r. *d.* *r.* *d.*
2d Simp. Queſt. 96 : 15 :: 120

The firſt *ſimple Queſtion* is *Inverſe*, the ſecond *Direct*, the *Mixt Queſtion* is *Inverſe* and *Direct*.

So the ſame *Mixt Queſtion* may be *Direct* and *Inverſe*, or *Inverſe* and *Direct*, according as the *firſt Queſtion* is *Direct* or *Inverſe*.

RULE IV. To work the *Direct* Rule.
Multiply the two firſt terms for a Diviſor, *and the three laſt for a* Dividend, *the* Quot *gives the Anſwer.*

RULE V. To work the *Inverſe* Rule.
Multiply the two laſt terms for a Diviſor, *and the three firſt for a* Dividend.

RULE VI. To work the *Direct* and *Inverſe* Rule.

Multiply the firſt and laſt terms for a Diviſor, *and the middle three for a* Dividend.

RULE VII. To work the *Inverſe* and *Direct* Rule.

Multiply the ſecond and fourth terms for a Diviſor, *and the remaining three terms for a* Dividend, *the* Quot *gives the Anſwer.*

Follows the operation of the three preceeding *Examples.*

Pr.	*D.*	*Int.*		*Pr.*	*D.*	*Int.*
100	: 365	: 5	::	798	: 86	

```
     365          798
     100           86
    -----        -----
    36500         4788
                 6384
                 -----
                 68628
                     5
                 -----
       36500)343140(9 l.
             328500
             ------
              14640 rem.
                 20
              -----
       365|00)2928|00(8 8/365 s.
              2920
              ----
                08
```

The foregoing queſtion being *Direct*, I multiply the two firſt terms, 100 and 365, for a *Diviſor*, and the three laſt terms, *viz.* 798 by 86, and the *Product* 68628 by 5 for a *Dividend*, and find the *Anſwer* is 9 *l.* $8\frac{8}{365}$ *s.*

This queſtion being *Inverſe*, I multiply the three firſt terms, which give 1728, for a *Dividend*, and the two laſt, which give 24, for a *Diviſor*, and find the *Anſwer* 72 inches of Thickneſs.

```
 L.   br.   th.    l. br. th.
 12 : 12 : 12 :: 6 : 4
      12           4
      ——          ——
      24          24
     12
     ——
     144
      12
     ——
     288
    144
    ———
 24)1728(72
    168.
    ———
     48
     48
     ——
      0
```

The

This queſtion being *Direct* and *Inverſe*, I multiply the firſt and laſt terms (*viz.* 96 by 8) for a *Diviſor*, and the middle three for a *Dividend*, and find the *Anſwer*, 23 days, 3 hours.

```
R.   M.   D.     R.   M. D.
96 : 12 : 15 :: 120 : 8
 8                15
 —                —
768               60
                  12
                 ——
                1800
                 12
                 —
                 36
                18
               ———
          768)21600(28 days
               1536
               ———
               6240
               6144
               ———
                 96 rem.
                 24
                 ——
                384
               192
               ———
           768)2304(3 hours
               2304
               ———
                  0
```

Here the queſtion being *Inverſe* and *Direct*, I multiply the 2d and 4th terms (*viz.* 96×8) for a *Diviſor*, and the remaining three, (*viz.* 15×12×120) for a *Dividend*, and the *Anſwer* comes out the ſame as before, *viz.* 28 days, 3 hours.

```
 M.   R.     D.   M.  R.   D.
12 : 96  :  15 :: 8 : 120
      8     12
    ---    ---
    768     30
            15
           ---
           180
           120
           ---
            36
           18
          ----
     768)21600(28 days
          1536
          ----
          6240
          6144
          ----
            96 rem.
            24
          ----
           384
          192
          ----
      768)2304(3 hours
          2304
          ----
             0
```

Every queſtion in the *compound Golden Rule* becomes a queſtion in the *ſimple Rule Direct*, thus,

State the two ſimple queſtions, into which the compound queſtion is reſolved, ſo as the number, which is of the ſame ſpecies with the number ſought, may ſtand in the third place, as before directed; then ſay, As the product of the Antecedents of the two firſt Ratios, is to the product of their Conſequents; ſo is the number which is of the ſame ſpecies with the number ſought, to the anſwer of the queſtion.

N. B. The alternate of this Proportion plainly illuſtrates the four Rules formerly laid down for working the *compound Rule*.

I ſhall take the ſame *Examples* as formerly.

EXAMPLE I.

If 5 *l.* is the intereſt of 100 *l.* for 365 days, what is the intereſt of 798 *l.* for 86 days?

Pr.		*Pr.*		*Int.*	*Int.*
100	:	798	::	5	
D.		*D.*			
365	:	86	::		

365×100 : 798×86 :: 5 *to the anſwer.*

Or,

36500 : 68628 :: 5 *to the anſwer.*

Here, as above, having, in my firſt ſimple queſtion, ſet 5 of intereſt in the third place, becauſe 798, being more principal, requires more intereſt than 100, I ſet my leſſer number, 100, in the firſt place; the ſame way, 86 days requiring leſs intereſt than 365 days, I ſet my greater number,

ber, 365 days, in the firſt place, in my ſecond ſimple queſtion; then I multiply my two Antecedents, 365 and 100, for an Antecedent, and my two Conſequents, 798 and 86, for a Conſequent, and continue 5 as my third term ſtill: So my *compound Queſtion* is reduced to a *ſimple one*, thus, As 365×100 is to 798×86, ſo is 5 to a 4th *Proportional*, which is the anſwer of my queſtion; or actually multiplying the Antecedents for a ſimple Antecedent, and the Conſequents for a Conſequent, as 36500 to 68628, ſo is 5 to the anſwer of the queſtion.

N. B. This *Proportion* alternated ſtands thus, 36500 : 5 :: 68628 to the anſwer; ſo multiplying (by the general rule of *Proportion*) the two laſt terms, and dividing by the firſt, is the ſame thing as ſtating the five numbers as formerly directed, and multiplying the three laſt terms for a *Dividend*, and the two firſt for a *Diviſor*.

EXAMPLE II.

If 12 inches of length, and 12 of breadth, require 12 of thickneſs to make a ſolid foot, what will 6 of length, and 4 of breadth require?

L.		*L.*		*Th.*	*Th.*
6	:	12	::	12	*1ſt Simp. Queſt.*
br.		*br.*			
4	:	12	::		*2d Simp. Queſt.*
6×4	:	12×12	::	12	*to the anſwer.*
				Or,	
24	:	144	::	12	*to the anſwer.*

Here, in both ſimple queſtions, leſs gives more; ſo I ſet the leſſer number in the firſt place in both, and both are *direct*; the product of the Antecedents of the two firſt Ratios in the ſimple queſtions, is an Antecedent, the product of their Conſequents, is a Conſequent of a new Ratio, the third term continues the ſame; ſo 6×4 : 12×12 :: 12 to the anſwer, or 24 : 144 :: 12 to the anſwer of the queſtion.

This Proportion taken alternately ſtands thus, 24 : 12 :: 144 to the anſwer.

Here, multiplying the laſt two terms, and dividing by the firſt, is the ſame thing as ſtating the Queſtion as formerly, and multiplying the three firſt numbers for a *Dividend*, and the two laſt for a *Diviſor*, which is the rule for working the *Inverſe Compound Rule*.

EXAMPLE III.

If, in 15 days, 12 men work 96 roods of work, in what time will 8 men work 120 roods at that rate?

Here, having ſtated the two *ſimple Queſtions*, ſo as the number, which is of the ſame ſpecies with the number ſought, ſtands in the 3d place, the product of the Antecedents of the two firſt Ratios, is to the product of their Conſequents, as the third number is to the Anſwer.

R. *R.* *D.* *D.*
96 : 120 :: 15
M. *M.*
8 : 12 ::

96×8 : 120×12 :: 15 *D.*

This Proportion is the ſame, as multiplying the firſt and laſt terms for a *Diviſor*, and the middle three for a *Dividend* in the *Direct* and *Inverſe* Rule, or multiplying the ſecond and fourth terms for a *Diviſor*, and the remaining three for a *Dividend* in the *Inverſe* and *Direct* Rule.

Thus, *Reduction*, and the *Golden Rule*, both *ſimple* and *compound*, are reduced to the general Rule of *Proportion*, *viz.* Multiply the two laſt terms, and divide the product by the firſt, and you have a fourth proportional term.

The compound Rule of *Fellowſhip*, or Fellowſhip with time, is only as many operations of the Rule of *Proportion* as there are parties concerned.

The method of working is, Multiply each man's ſtock into his time, the ſum of the products is the firſt term of your proportion, the whole loſs or gain is the ſecond, and the ſeparate products make the third term in the ſeveral operations, for finding each man's ſhare of gain or loſs.

EXAMPLE I.

Three perſons, *A*, *B* and *C*, hire a paſture for 2[illegible] *l.* *A* puts in 40 cows for 4 months, *B* 30 cows for 2 months, and *C* 36 for 5 months what ſhare of the rent muſt each perſon pay?

A $40\times4=160$, 3d term for *A*'s ſhare.
B $30\times2=\ 60$, 3d term for *B*'s ſhare.
C $36\times5=180$, 3d term for *C*'s ſhare.

400, 1ſt term.

Here 400, the ſum of the products, is the firſt term of the ſeveral proportions, the whole rent, 24 *l.* is the ſecond, and the third term in each proportion is as before ſtated: So the operation will ſtand thus,

```
  L.   A's ſhare.
400 : 24 ::   160
               24
              ——
               64
              32
           ——————
4|00)38|40( 9 l.
     36
     ——————
       240
        20
    ———————
 4|00)48|00(12 s.
      4
      ——
      08
       8
      ——
       0
```

```
   L.      B's ſhare.
400 : 24   ::   60
      60
   ————————
4|00)14|40(3 l.
     12
   ————————
     240
      20
4|00)48|00(12 s.
     4
     ——
     08
      8
     ——
      0
```

400

L. *C*'s *ſhare.*
400 : 24 :: 180
24
—
72
36
—
4|00)43|20(10 *l.*
4
—
0320
20
—
4|00)64|00(16 *s.*
4
—
24
24
—
0

A's ſhare of rent--9 *l.* 12 *s.*
B's ſhare —— 3 *l.* 12 *s.*
C's ſhare —— 10 *l.* 16 *s.*
—
Proof 24 *l.* 00 *s.*

EXAMPLE. II.

A and *B* put in ſtocks for trade, *A* puts in 40 *l.* and at 4 months end takes out 10 *l.* but 2 months after puts in 30 *l.* *B.* puts in 50 *l.* and at the end of 3 months puts in 20 *l.* more; at the end of 8 months they have gained 18 *l.* what is each man's ſhare of the gain?

A hath 40 *l.* then 30 *l.* then 60 *l.*
for 4 *mon.* 2 *mon.* 2 *mon.*

A in all 160 + 60 + 120 = 340

B hath 50 *l.* then 70 *l.*
for 3 *mon.* 5 *mon.*

B in all 150 + 350 - - - = 500

A and *B* together - - - - - - - 840

Here *A* has 40 *l.* in for 4 months, which produces 160, having then taken out 10 *l.* he has only 30 *l.* for the 2 following months, which produces 60 *l.* having then added 30 *l.* he has 60 *l.* for the 2 months that remain of the 8 months, which produces 120; ſo *A* has in all 160 + 60 + 120 = 340.

Again, *B* has 50 *l.* for 3 months, which produce 150; to this 50 he, at the end of 3 months, adds 20 *l.* ſo he has 70 *l.* for the laſt 5 of the 8 months, which produces 350; ſo *B* has in all 150 + 350 = 500.

Then *A* and *B* together have 340 + 500 = 840.

Here, as 840 is to the whole gain 18 *l.* ſo is 340 to *A*'s ſhare of the gain 7 *l.* 5 *s.* 8 *d.* 2 $\frac{24}{84}$ *f.*

N. B. In my firſt *Diviſion* the cypher on the right hand of the *Dividend*, cancelling the cypher on the right hand of the *Diviſor*, I regard it no more, but divide by 84 in the reſt of my *Diviſions*, for finding the Shillings, Pence and Farthings.

840 : 18 :: 340
18
———
272
34
———
84|0)612|0(7 *l.*
588
———
24 *rem.*
20
———
84)480(5 *s.*
420
———
60 *rem.*
12
———
84)720(8 *d.*
672
———
48 *rem.*
4
84)192(2 $\frac{24}{84}$ *f.*
168
———
24

Here again 840 is to 18, as 500 to *B*'s ſhare of gain 10 *l.* 14*s.* 3 *d.* 1 $\frac{60}{84}$ *f.*

	L.	*s.*	*d.*	*f.*
A's gain	7	5	8	$2\frac{24}{84}$
B's —	10	14	3	$1\frac{60}{84}$
Proof	18	00	0	0

The two ſhares added together, make juſt 18 *l.* which proves the work.

N. B. For the addition of the Fractions of a Farthing, we muſt refer the reader to the ſubſequent doctrine of Fractions.

840 : 18 :: 500
500
84|0)900|0(10 *l.*
84
60 *rem.*
20
84)1200(14 *s.*
84
360
336
24 *rem.*
12
48
24
84)288(3 *d.*
252
36
4
84)144(1 $\frac{60}{84}$ *f.*
84
60

Ex-

Examples of the Compound Rule of Three.

Queſtion 1. If the carriage of 20 packs, for 136 miles, coſt 16 *l.* what will the carriage of 12 packs be for 28 miles, at the ſame rate? *Anſwer*, 1 *l.* 19 *s.* 6 *d.* 1 $\frac{56}{136}$, or $\frac{7}{17}$ *f.*

Queſt. 2. If 60 hundred weight coſt 14 *l.* 10 *s.* for being carried 20 miles, what will 15 hundred weight coſt to be carried 30 miles at that rate? *Anſ.* 5 *l.* 8 *s.* 9 *d.*

Queſt. 3. If 48 *s.* be wages for 6 men for 6 days, how much wages muſt 48 men have for 30 days at the ſame rate? *Anſ.* 96 *l.*

Queſt. 4. If 18 roods of ditching be done by 3 men in 6 days, how many roods will be wrought by 8 men in 4 days at the ſame rate of working? *Anſ.* 32 roods.

Queſt. 5. If 10 canons ſpend 40 barrels of pouder in 1 day, how many barrels will 24 cannon ſpend in 30 days at the ſame rate? *Anſwer*, 2880 barrels.

Queſt. 6. If 100 *l.* yield 5 *l.* intereſt in 1 year, how much intereſt will 850 *l.* yield in 3 years and 8 months? *Anſ.* 155 *l.* 16 *s.* 8 *d.*

Queſt. 7. What principal will yield 155 *l.* 16 *s.* 8 *d.* intereſt in 3 years and 8 months at the rate of 5 *per Cent.* intereſt *per Annum*? *Anſ.* 850 *l.*

Queſt. 8. In what time will 850 *l.* principal yield 155 *l.* 16 *s.* 8 *d.* intereſt, at the rate of 5 *per Cent. per Annum*? *Anſ.* 3 years and 8 months.

Queſt. 9. An uſurer put out 455 *l.* principal at intereſt, and after it had continued for 3 years and 4 months, he received for principal and intereſt together, 576 *l.* 6 *s.* 8 *d.* at what rate *per Cent. per Annum* did he receive intereſt? *Anſ.* 8 *per Cent.*

Queſt. 10. On the 25th day of *March* 1732, a certain perſon lent on a mortgage 760 *l.* and upon the 25th of *May* 1738, he received for intereſt thereof 281 *l.* 4 *s.* I demand at what rate *per Cent. per Annum* his money was lent? *Anſ.* At the rate of 6 *per Cent.*

Queſt. 11. If the carriage of 60 hundred weight for 20 miles coſt 14 *l.* 10 *s.* what weight ought to be carried 30 miles for 5 *l.* 8 *s.* 9 *d.* at that rate? *Anſ.* 15 hundred.

Queſt. 12. If 60 hundred be carried 20 miles for 14 *l.* 10 *s.* how many miles ought 15 hundred to be carried for 5 *l.* 8 *s.* 9 *d.* at the ſame rate? *Anſ.* 30 miles.

Queſt. 13. If 8 men mow 112 acres of graſs in 14 days, how many men can mow 2000 acres at the ſame rate, in 10 days? *Anſ.* 200 men.

Queſt. 14. If 2000 acres of graſs are mowen by 200 men in 10 days, how many acres, at that rate, will 8 men mow in 14 days? *Anſ.* 112 acres.

Queſt. 15. If 200 men can mow 2000 acres of graſs in 10 days, how many days will 8 men take to mow 112 acres at that rate? *Anſ.* 14 days.

Queſt. 16. If 170 buſhels of wheat ſerve 680 men for 6 days, how much will ſerve 79200 men, at that rate, for 16 days? *Anſ.* 6600 quarters.

N. B. 8 buſhels make 1 quarter.

Queſt. 17. If 2 horſes eat 8 buſhels of oats in 16 days, how many horſes will eat up 3000 quarters in 24 days, at the ſame rate? *Anſ.* 4000 horſes.

Queſt. 18. If 275 men caſt a trench of 250 roods in 12 hours, how many men, at that rate, will caſt a trench of 880 roods in 8 hours? *Anſ.* 1452 men.

Queſt. 19. If a wall of 12 feet long and 12 feet broad, is alſo to be made 12 feet high, what muſt be the height of a wall of 4 feet long, and 6 feet broad, ſo as it may be of the ſame contents with the former? *Anſ.* 72 feet high.

Queſt. 20. If a veſſel, holding 700 gallons of liquor, be 3 feet deep, 5 feet long, and 4 feet broad, what muſt be the breadth of a veſſel to hold 700 gallons, when it is to be 5 feet deep and 6 feet long? *Anſ.* 2 feet broad.

Queſtions for the Practice of FELLOWSHIP.

Queſtion 1. Three traders *A*, *B* and *C*, have dealt in company; *A* put into the common ſtock, upon the 1ſt of *January*, 1000 *l.* *B*, upon the 5th, put in 500 *l.* and *C*, upon the 12th of *July*, put in 800 *l.* At the year's end they balance, and find 180 *l.* gained, what is each partner's ſhare of the gain? *Anſ.* *A*'s 96 *l.* 1 *s.* 4 *d.* *B*'s 47 *l.* 10 *s.* 1 *d.* 2 *f.* and *C*'s 36 *l.* 8 *s.* 6 *d.* 2 *f.*

Queſt. 2. Three perſons, *A*, *B* and *C*, were partners in trade, *A* depoſited 1200 *l.* upon the 1ſt of *January*, and, upon the 1ſt of *April*, he withdrew 200 *l.* *B.* put in 600 *l.* upon the 1ſt of *March*; and upon the 1ſt of *Auguſt* added 250 *l.* more: *C* put in 500 *l.* upon the 1ſt of *July*, and the 1ſt of *October* he took out 100 *l.* At the year's end they balanced accompts, and find they had gained 300 *l.* what muſt each partner's ſhare the gain be? *Anſ.* *A*'s 167 *l.* 12 *s.* 6 *d.* 2 *f.* *B*'s 96 *l.* 9 *s.* 0 *d.* 2 *f.* and *C*'s 35 *l.* 18 *s.* 5 *d.*

Queſt. 3. Three merchants, *A*, *B* and *C*, join their ſtocks together, and make up 12400 *l.* with which they traded, and gained 2480 *l.* of which *A* gets 686 *l.* *B* 870 *l.* *C* 924 *l.* I demand what each partner put in at firſt?

Anſwer,
A put in 3430 *l.*
B —— 4350 *l.*
C —— 4620 *l.*

L. 12400 the whole ſtock.

The *Golden Rule Compound* may extend to ſeven numbers given, and an eighth number ſought, or to nine numbers given, and a tenth ſought, or to eleven numbers given, and a twelfth ſought, &c. which Rules we may call *the Rule of Seven*, *the Rule of Nine*, &c.

The method of ſtating the Queſtion depends on the ſame principles, and the manner of proceeding is the ſame, as in the *Rule of Five*, thus,

Set that number which is of the ſame ſpecies or kind with the number ſought, in the middle, the ſeveral numbers that give it, on its left hand, and the numbers which afford the number ſought, on its right hand, in the ſame order with thoſe on the left hand, *viz.* the firſt on the left hand, and the firſt on the right hand of the ſame ſpecies; the ſecond on the left hand of the middle term, and the ſecond on its right hand of the ſame ſpecies, *&c.*

N. B. Every compound queſtion may be reſolved into as many ſimple queſtions, as there are terms on the left hand of the middle term.

RULE, To work any compound queſtion. *Having reſolved your compound queſtion into all its ſimple ones, ſo as the middle term of the compound queſtion ſhall be the third term in each ſimple queſtion, multiply all the antecedents of the firſt ratios continually for an antecedent, and all their conſequents for the conſequent of a new ratio, and ſay, As this new antecedent is to its conſequent, ſo is the common third term to a fourth proportional, which is the Anſwer of the queſtion.*

What has hitherto been ſaid, may ſerve as a general preparation for the practice of the forementioned Rules of *Seven*, *Nine*, *Eleven*, or more, if the queſtion affords them; we ſhall now begin with a queſtion of the *Rule of Seven.*

If 15 men eat 13 Shillings worth of bread in 6 days, when wheat is at 12 Shillings *per* boll, what will be the price of bread to ſerve 30 men for 12 days, at the ſame rate of eating, when wheat is at 10 Shillings *per* boll?

M. *d.* *ſh.* per *b.* *ſh. br.* *m.* *d.* *ſh.* p. *boll.*
15 : 6 : 12 : 13 30 :: 12 : 10 *Anſ.*

Here I ſet the number, which is of the ſame ſpecies with the number ſought, (*viz.* 13 Shillings worth of bread) in the middle, the three numbers that give it (*viz.* 15 men, 6 days, and 12 Shillings *per* boll) on its left hand, and, in the ſame order, on its right hand, the three numbers that give the term ſought, *viz.* 30 men, 12 days, and 10 Shillings *per* boll, and the *Anſwer* comes in the eight place.

Next, I reduce my compound queſtion to its three ſimple ones.

M. *m.* *ſh. br.*
1*ſt Simp. Queſt.* 15 : 30 :: 13 : to a 4th.
d. *d.*
2*d Simp. Queſt.* 6 : 12 ::
s. p. *b.* *s.*p.*b.*
3*d Simp. Queſt.* 12 : 10 ::

Then I ſay 15×6×12 : 30×12×10 :: 13 : to the *Anſ.*

Or actually multiplying as follows.

```
                                         15      12
                                          6      30
                                         ——      ——
                                         90     360
                                         12      10
                                       ————    ————
                                       1080    3600
                                       ————    ————
1080 : 3600 :: 13         Anſ. 2 l. 3 s. 4 d.
        13
       ——
       108
       36
10|80)4680|0(43 s. = 2 l. 3 s.
       432
       ———
        360
        324
        ———
         36 rem.
         12
         ——
         72
        36
        ——
  108)432(4 d.
      432
      ———
```

Queſt. 2. If 18 roods of ditching be wrought by 3 men in 16 days, when the day is 15 hours long, how much will be done by 8 men in 4 days, at the ſame rate of working, when the day is 9 hours long? *Anſ.* $7\frac{1}{5}$ roods.

Queſt. 3. If 15 men eat 13 Shillings worth of bread in 6 days, when wheat is at 12 Shillings *per* boll, how many days will 30 men take, at that rate, to eat 43 Shillings 4 Pence worth of bread, when wheat is at 10 Shillings *per* boll? *Anſ.* 12 days.

Example of the Rule of Nine.

If 6 yards of Linen be worth 12 yards of Drugget, or 4 yards of Cloth, or 30 Yards of Muſline, or 60 pounds of Tobacco, how many yards of Linen will be worth 16 yards Drugget, 20 yards Cloth, 9 yards Muſline, or 360 pounds of Tobacco?

Dr. cl. muſl. lb.t. y.lin dr. cl. muſl. lb.t.
12 : 4 : 30 : 60 : 6 :: 16 : 20 : 9 : 360

Here I ſet the yards of Linen, which are of the ſame ſpecies with the number ſought in the middle, the four terms that give it on its left hand, and on its right hand, in the ſame order, the four that afford the *Anſwer*.

Next I ſtate my ſimple queſtions, thus,

Y.dr. *y.dr.* *y.lin.*
1ſt Simp. Queſt. 12 : 16 :: 6
y.cl. *y.cl.*
2d Simp. Queſt. 4 : 20 ::
y.muſ. *y.muſ.*
3d Simp. Queſt. 30 : 9 ::
lb.tob. *lb.tob.*
4th Simp. Queſt. 60 : 360 ::

12	16
4	20
48	320
30	9
1440	2880
60	360
86400	1728
	864
	1036800

Then I ſay, As 12×4×30×60 : 16×20×9×360 :: 6 to the *Anſwer*.

Or multiplying the terms, as

86400 : 1036800 :: 6 to the *Anſwer.*

6

864|00)62208|00(72 *yds.* *Anſ.*

6048·

1728

1728

0000

Example of the Rule of Eleven.

If 12 men build a wall 30 feet long, 6 high, and 3 thick, in 15 days, when the day is 12 hours, in how many days will 60 men build a wall 300 feet long, 8 high, and 6 thick, when they work 8 hours each day?

I ſtate the queſtion, according to the Rule already laid down, thus,

Men.	*len.*	*ht.*	*th.*	*h.*	*d.*	*m.*	*len.*	*ht.*	*th.*	*h.*
12 :	30 :	6 :	3 :	12 :	15 ::	60 :	300 :	8 :	6 :	8

Then I ſtate my ſimple queſtions, thus,

	M.	*m.*	*d.*
1ſt Simp. Queſt.	60 :	12 ::	15
	len.	*len.*	
2d Simp. Queſt.	30 :	300 ::	
	ht.	*ht.*	
3d Simp. Queſt.	6 :	8 ::	
	th.	*th.*	
4th Simp. Queſt.	3 :	6 ::	
	hours.	*hours.*	
5th Simp. Queſt.	8 :	12 ::	

60	12
30	300
1800	3600
6	8
10800	28800
3	6
32400	172800
8	12
259200	3456
Antecedent.	1728
	2073600
	Conſequent.

Then as 60×30×6×3×8 : 12×300×8×6×12 :: 15 to the *Anſwer*.

Or 259200 : 2073600 :: 15 to the *Anſwer*.
15

103680
20736

2592|00)311040|00(120 *days*.
2592··

5184
5184

00 *rem*.

This work might be ſhortened, by leaving out of both *Antecedent* and *Conſequent* the equal multipliers 6 and 8, thus,

60×30×3 : 12×300×12 :: 15

60 12
30 300

1800 3600
3 12

5400 72
Antecedent. 36

43200
Conſequent.

Or 5400 : 43200 :: 15
15

2160
432

54|00)6480|00(120 *days.*
54··

108
108

0

Example for Practice.

If 100 *lb.* of *Venice* weigh 70 *lb.* of *Lyons*, and 120 *lb.* of *Lyons* weigh 100 *lb.* of *Roan*, and 80 *lb.* of *Roan* 100 *lb.* of *Toulouſe*, and 100 *lb.* of *Toulouſe* 74 *lb.* of *Geneva*, how many *lb.* of *Geneva* will 100 *lb.* of *Venice* weigh?

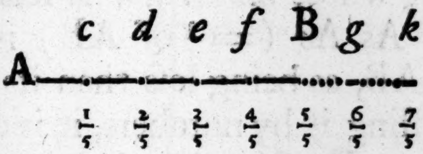
Anſwer $53\frac{92}{96}$ *lb.*

FRACTIONS.

FRACTIONS.

DEFINITION I.

A **Fraction** is a part or parts of Unity, or any whole.



Thus, ſuppoſing the line AB to repreſent any whole, as one ell, one mile, or one pound, it is Unity, or one of its kind; and conceiving that whole AB to be divided into any number of equal parts (as 5) at the point *c*, *d*, *e*, *f*, it is plain any of theſe parts as A*c* is one fifth part of the whole AB, or it is one part of which that whole contains five, and being a part of the whole, it is a Fraction of that whole: Again, A*d* is two fifth parts of AB, or two parts of which that whole (AB) contains five, and being parts of the whole, it is a Fraction of that whole.

Every Fraction is neceſſarily expreſſed by two numbers, the one written, for ordinary, above, the other below a line: Thus, the Fraction $Ae=\frac{3}{5}$ of the whole AB.

The number below the line ſhows into how many parts the whole is divided, and conſequently expreſſes one whole; it denominates, or gives name to the ſeveral Fractions ariſing from the diviſion of the whole into that number of parts; and is therefore called the *denominator* of the

Fraction. The number above the line ſhows how many of theſe parts the Fraction contains, and is thereſore called its *numerator*; ſo the *numerator* is the Fraction, the *denominator* only the name of the Fraction.

Definition II. A *proper Fraction* is leſs than its whole, or whoſe *numerator* is leſs than its *denominator*: As A*e* ($=\frac{3}{5}$ of AB) is properly a Fraction of AB, as being leſs than its whole; and in the expreſſing it by numbers, it is eaſily known to be a proper Fraction by the *numerator* 3 being leſs than the *denominator* 5.

Definition III. An *improper Fraction* is equal to, or greater than its whole, or whoſe *numerator* is equal to, or greater than its *denominator*; as AB$=\frac{5}{5}$, or A*k*$=\frac{7}{5}$.

AB is properly one whole; but as $\frac{5}{5}$ has a *numerator* and a *denominator*, and expreſſes parts of a whole, it is a Fraction, but improperly ſo. The ſame way A*k* is one whole, and $\frac{2}{5}$ of another whole; but as $\frac{7}{5}$ expreſſes parts of a whole, it is a Fraction, tho' improperly ſuch.

Definition IV. A *ſimple Fraction* has but one *numerator* and one *denominator*; as $\frac{2}{3}$, or $\frac{7}{5}$.

Definition V. A *compound Fraction* joins two or more ſimple Fractions with the particle *of*, and is the fraction of a Fraction; as $\frac{2}{3}$ of $\frac{4}{5}$.

Definition VI. A *mixt number* is compoſed of an Integer and a Fraction; as $8\frac{2}{3}$.

Re-

Reduction of FRACTIONS.

RULE I. To reduce *improper Fractions* to Integers, or mixt numbers.

Divide the numerator *by the* denominator, *the* Quot *gives Integers*; *the* Remainder *is the* numerator *of the Fraction to be annexed, whoſe* denominator *is the ſame as before.*

EXAMPLE. I.

In *Example* 1ſt I divide the *numerator*, 776, by the *denominator* 8, and the *Quot* comes out juſt 97 wholes.

$\frac{776}{8}=97$

```
8)776(97
  72.
  —
   56
   56
   —
    0
```

EXAMPLE II.

$\frac{288}{23}=12\frac{12}{23}$

```
23)288(12
   23.
   —
    58
    46
    —
    12
```

The reaſon of the Rule is plain; for the *denominator* of every Fraction being one whole, and the *numerator* being the whole content of the Fraction, to ask how many wholes the Fraction contains, is to ask, How often the *denominator* is contained in the *numerator*, *i.e.* to divide the *numerator* by the *denominator*.

The preceeding Rule may be otherwiſe accounted for, thus,

As, in the Definition of an *improper Fraction*, $\frac{5}{5}=1$ whole, it is plain $\frac{10}{5}=2$ wholes, $\frac{15}{5}=3$ wholes, &c. ſo in the Fraction $\frac{5}{5}$, dividing the *numerator* 5 by the *denominator* 5, the *Quot* gives 1 whole, as the value of the Fraction; and in the Fraction $\frac{10}{5}$, dividing the *numerator* 10 by the *denominator* 5, the *Quot* gives 2 wholes, the value of the Fraction, &c. The ſame way A*k*$=\frac{7}{5}$, and dividing the *numerator* 7 by the *denominator* 5, the *Quot* gives 1 whole, and 2 fifths (or $\frac{2}{5}$) over; ſo $\frac{7}{5}=1\frac{2}{5}$.

RULE II. To reduce mixt numbers to *Improper Fractions.*

> *Multiply the* Integer *by the* Denominator; *to the* Product *add the* Numerator; *that ſum gives the* Numerator *of the* Improper Fraction, *whoſe* Denominator *is the ſame as before.*

	12		48
$12\frac{12}{23}=\frac{288}{23}$	23	$48\frac{18}{25}=\frac{1218}{25}$	25
	36		240
	24		96
	276		1200
	+12		+18
	288		1218 *Numerator.*

This, being the reverſe of the former Rule, is no more than the common proof of *Diviſion.*

RULE III. To reduce Integers to Fractions of a given *Denominator.*

Multiply the Integer *by the given* Denominator, *for the* numerator *of the* Fraction.

Thus, if I am to reduce 36 to a Fraction, whoſe *denominator* is 4, I multiply 36 by 4, and find 144 to be my *numerator.*

$$36 = \tfrac{144}{4} \qquad \begin{array}{r} 36 \\ 4 \\ \hline 144 \end{array}$$

The reaſon is plain; for if the *Integer* (36) was ells of cloth, to be reduced to quarters, or fourth parts, it is plain, from common *Reduction*, that I muſt multiply 36 by 4, to find the quarters, or fourth parts in that *Integer.*

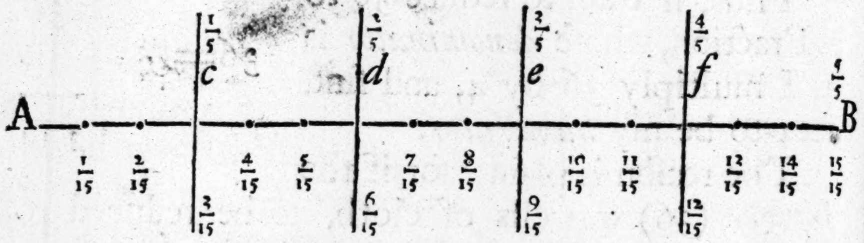
COROLARY.

Hence it is plain, that to reduce *Integers* to the form of a Fraction, is to make Unity the *Denominator*; for making 8 the *Integer*, it is plain, that, multiplying the *Integer* by the *Denominator*, 1, gives the ſame 8 for the *Numerator* of the Fraction.

$$8 = \tfrac{8}{1}$$

THEOREM.

If both *Numerator* and *Denominator* of any Fraction, are multiplied by one and the ſame number, the two *Products* make a new Fraction equal in value to the former. Thus, if I multiply both *Numerator* and *Denominator* of $\frac{2}{5}$ by 3 I have $\frac{6}{15}=\frac{2}{5}$



For, dividing the whole AB into 5 equal parts at the points *c*, *d*, *e*, *f*, it is plain A*d*$=\frac{2}{5}$ of AB, and dividing each of theſe equal parts into 3 equal parts, (*viz.* the number of Units in the common multiplier) the whole AB is divided into 15=3×5, and A*d* into 6=3×2.

So $\frac{3\times2}{3\times5}=\frac{6}{15}=Ad=\frac{2}{5}$

COROLARY.

If both *Numerator* and *Denominator* of any Fraction are divided by the ſame number, the two *Quots* make a new Fraction equal in value to the former. Thus, dividing both *Numerator* and *Denominator* of $\frac{6}{15}$ by 3, we have $\frac{6}{15}=\frac{2}{5}$

DEFINITION.

Two Fractions are ſaid to be equal, when the *Denominator* of the one is to its *Numerator* as the *Denominator* of the other is to its *Numerator*;

Thus $\frac{2}{5}=\frac{6}{15}$ where 5 : 2 :: 15 : 6.

PROBLEM.

To find a Fraction equal to a given Fraction, and having a given *Denominator.*

> *Multiply the* Numerator *by the given* Denominator, *and divide the* Product *by the* Denominator *of the Fraction, the* Quot *gives the* Numerator *of the new Fraction.*

EXAMPLE.

To find a Fraction equal to $\frac{3}{4}$, whoſe *Denominator* is 36.

$\frac{3}{4}=\frac{27}{36}$

```
4 : 3 :: 36
          3
         ——
     4)108(27 Numerator.
        8
        —
        28
        28
        ——
         0
```

Here 4, 3 and 36 being given, the 4th *Proportional* is found by dividing the *Product* of the two laſt terms by the firſt, which is the ſame thing with the above *Problem.*

R U L E IV. To reduce *compound Fractions* to *ſimple Fractions*;

Multiply the Numerators *continually for the* Numerator *of the ſimple Fraction, and the* Denominators *for its* Denominator, *thus*,

$\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$

$$\begin{array}{ll} 2 & 3 \\ 4 & 5 \\ \hline 8 \textit{ Num.} & 15 \textit{ Denom.} \end{array}$$

And $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{9}=\frac{56}{135}$

$$\begin{array}{ll} 2 & 3 \\ 4 & 5 \\ \hline 8 & 15 \\ 7 & 9 \\ \hline 56 \textit{ Numer.} & 135 \textit{ Denom.} \end{array}$$

The reaſon is plain; for in *Example* I. *viz.* $\frac{2}{3}$ of $\frac{4}{5}$, by the preceeding *Theorem* $\frac{4}{5}=\frac{12}{15}$, and $\frac{1}{3}$ of $\frac{12}{15}=\frac{4}{5}$, conſequently $\frac{2}{3}$ of $\frac{12}{5}=\frac{8}{15}=\frac{2\times4}{3\times5}$

R U L E V. To value Fractions in the known parts of the *Integer*;

Multiply the Numerator *by the number of known parts contained in the* Integer, *and divide the* Product *by the* Denominator, *the* Quot *gives the* Anſwer.

Thus, if the value of $\frac{3}{4}$ of a Pound is required, I multiply the *Numerator*, 3, by 20, the number of Shillings in a Pound, and divide the *Product*, 60, by 4, the the *Denominator*, and the *Quot* gives 15 Shillings.

l. *s.*
$\frac{3}{4}$ = 15

3
20
—
4)60(15 *s.*
4
—
20
20
—

Here, having reduced $\frac{9}{16}$ of a Pound to 11 Shillings, 4 remain, which make $\frac{4}{16}$ of a Shilling; ſo I multiply the *Numerator* 4 by 12, the *d.* in a Shilling, and divide the *Product* by the *Denominator* 16, the *Quot* gives 3 Pence.

l. *s.* *d.*
$\frac{9}{16}$ = 11 3

9
20
16)180(11$\frac{4}{16}$ *s.*
16
—
20
16
—
4 *rem.*
12
—
16)48(3 *d.*
48
—
0

Year. *m.* *w.* *d.*

$\frac{8}{14} = 7 : 1 : 5$

$$\begin{array}{l}
\quad\;\; 8 \\
\quad\;\; 13 \\
\hline
14)104(7\tfrac{6}{14}\ m. \\
\quad\;\; 98 \\
\hline
\quad\;\; 6 \\
\quad\;\; 4 \\
\hline
14)24(1\tfrac{10}{14}\ w. \\
\quad\;\; 14 \\
\hline
\quad\;\; 10 \\
\quad\;\; 7 \\
\hline
14)70(5\ d. \\
\quad\;\; 70 \\
\hline
\quad\;\; 0
\end{array}$$

The reaſon is plain; for ſeeing a Shilling is $\frac{1}{20}$ of a Pound, to reduce $\frac{3}{4}$ of a Pound to Shillings, is to reduce $\frac{3}{4}$ to a Fraction, whoſe *Denominator* is 20, and 4 : 3 :: 20 to the *Numerator*, or number of Shillings in the Fraction.

L E M-

LEMMA.

To find the greateſt common *Diviſor* to two given numbers;

Divide the greater number by the leſſer, and the Diviſor *by the* Remainder, *and ſo on continually till* Unity *or* 0 *remains. If* 1 *remains, the numbers have no common* Diviſor: *If* 0 *remains, the laſt* Diviſor *is their greateſt common* Diviſor.

EXAMPLE I.

I require the greateſt common *Diviſor* to 784 and 952.

Here I have continued my *Diviſion* till 0 remains; ſo the laſt *Diviſor*, 56, is the greateſt common *Diviſor* to 784 and 952.

```
784)952(1
    784
    ---
    168)784(4
        672
        ---
        112)168(1
            112
            ---
             56)112(2
                112
                ---
                  0
```

EX-

EXAMPLE II.

I demand the greateſt common *Diviſor* to 468 and 846.

Anſwer 18.

```
468)846(1
    468
    ———
    378)468(1
        378
        ———
         90)378(4
            360
            ———
             18)90(5
                90
                ——
                 0
```

EXAMPLE III.

I require the greateſt common meaſure to 147 and 323.

Here 1 remains; ſo the two numbers have no common *Diviſor*.

```
147)323(2
    294
    ———
     29)147(5
        145
        ———
          2)29(14
            2
            —
            09
             8
             —
             1
```

See the demonſtration in the 2d propoſition of the 7th Book of *Euclid*.

RULE VI. To abbreviate Fractions, *i. e.* to reduce them to their loweſt terms;

Divide both Numerator *and* Denominator *by their greateſt common* Diviſor, *the two* Quots *make the new* Fraction.

Thus $\frac{784}{952} = \frac{14}{17}$

```
56)784(14    56(952(17
   56·          56·
   ——           ——
   224          392
   224          392
   ——           ——
    0            0
```

It follows from the *Corollary* of the preceeding *Theorem.*

Or thus,

Divide both Numerator *and* Denominator *by their leaſt common* Diviſor, *and you have your Fraction in lower terms; after the ſame manner reduce that new Fraction to lower terms, and ſo on continually till no common* Diviſor *is found, and you have your Fraction in its loweſt terms.*

$\frac{2)468}{2)846} = \frac{3)234}{3)423} = \frac{3)78}{3)141} = \frac{26}{47}$

N. B. In this Fraction $\frac{468}{846}$, becauſe 8 and 6 the Units of both *Numerator* and *Denominator*, are even numbers, I know 2 meaſures both; ſo, dividing by 2, I have a new Fraction $\frac{234}{423}$, where 3, the Units of the *Denominator*, is an odd number, ſo 2 cannot meaſure both *Numerator* and *Denominator*, I try 3, it meaſures both, and affords a new Fraction $\frac{78}{141}$: Here again 3 meaſures both, and gives a new Fraction $\frac{26}{47}$; here 3 cannot meaſure both, and it is in vain to try 2; for it could not meaſure $\frac{234}{423}$: If 2 cannot meaſure $\frac{26}{47}$, far leſs can 4, which is a *multiple* of 2, becauſe neither 2 nor 3 can meaſure $\frac{26}{47}$, it is plain 6, being a *multiple* of 2 and 3, cannot meaſure both; by tryal I find 7 cannot, as little can 8, being a *multiple* of 2, or 9 being a *multiple* of 3; ſo $\frac{26}{47}$ is the loweſt $\frac{468}{846}$ can be reduced to.

N. B. 2. The greateſt common *Diviſor* to two given numbers is the *Product* ariſing from the continual multiplication of all their leaſt common *Diviſors*; thus, the greateſt common meaſure to 468 and 846 is $18=2\times3\times3$.

N. B. 3. If 5 is on the right hand of both *Numerator* and *Denominator*, or 5 in the one and 0 in the other, 5 meaſures both; ſo $\frac{5)\,125}{5)\,345}=\frac{25}{69}$, and $\frac{5)\,345}{5)\,800}=\frac{69}{160}$.

N. B. 4. If there are cyphers on the right hand of both *Numerator* and *Denominator*, cut off an equal number of cyphers from both, and and you have your Fraction in lower terms; thus, $\frac{8|00}{60|00}=\frac{8}{60}$; for it is the ſame with dividing both 800 and 6000 by 100.

RULE VII. To reduce Fractions of different *Denominators* to a common *Denominator*;

Multiply the Denominators *continually for the common* Denominator.

Multiply each Numerator *into all the* Denominators, *except its own, for its correſponding* Numerator.

So $\frac{2}{3}+\frac{4}{5}=\frac{10}{15}+\frac{12}{15}$

$$\begin{array}{ccc} 5 & 2 & 4 \\ 3 & 5 & 3 \\ \hline 15\ \textit{c.d.} & 10\ \textit{num.} & 12\ \textit{num.} \end{array}$$

The reaſon is plain; for both *Numerator* and *Denominator* of $\frac{2}{3}$ is multiplied by the ſame number 5; ſo $\frac{2}{3}=\frac{10}{15}$, and both *Numerator* and *Denominator* of $\frac{4}{5}$ is multiplied by 3, ſo $\frac{4}{5}=\frac{12}{15}$, conſequently $\frac{2}{3}+\frac{4}{5}=\frac{10}{15}+\frac{12}{15}$ by the above *Theorem.*

EXAMPLE II.

$\frac{2}{3}+\frac{4}{5}+\frac{7}{9}=\frac{90}{135}+\frac{108}{135}+\frac{105}{135}$

Here both *Numerator* and *Denominator* of $\frac{2}{3}$ are multiplied 5×9, or 45, ſo $\frac{2}{3}=\frac{90}{135}$, both *Numerator* and *Denominator* of $\frac{4}{5}$, are multiplied by 3×9 or 27, ſo $\frac{4}{5}=\frac{108}{135}$, and both *Numerator* and *Denominator* of $\frac{7}{9}$ are multipled by 3×5, or 15; ſo $\frac{}{9}=\frac{105}{135}$

$$\begin{array}{cccc} 3 & 2 & 4 & 7 \\ 5 & 5 & 3 & 3 \\ \hline 15 & 10 & 12 & 21 \\ 9 & 9 & 9 & 5 \\ \hline 135\ \textit{c. d.} & 90\ \textit{n.} & 108\ \textit{n.} & 105\ \textit{n.} \end{array}$$

N. B. If the *Denominator* of one Fraction meaſures the *Denominator* of another Fraction, multiply both *Numerator* and *Denominator* of the Fraction that has the leſſer *Denominator*, by the number of times the greater *Denominator* contains the leſſer, and your Fractions are at a common *Denominator*;

Thus $\frac{2}{3}+\frac{5}{12}=\frac{8}{12}+\frac{5}{12}$

Here 12 contains 3 four times, ſo I multiply both 2 and 3 by 4, and I have $\frac{8}{12}=\frac{2}{3}$

And $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}=\frac{4}{8}+\frac{6}{8}+\frac{5}{8}$

Here 2 is four times in 8; ſo I multiply both 1 and 2 by 4, and $\frac{1}{2}=\frac{4}{8}$: Again, 4 is twice in 8, ſo I multiply both 3 and 4 by 2, and I have $\frac{6}{8}=\frac{3}{4}$; the reaſon is plain from the above *Theorem.*

N. B. Becauſe only things of the ſame ſpecies or kind can be added, ſubtracted, &c. before you begin to add or ſubtract, &c. Fractions,

Obſerve 1. If you have Integers mixt with your Fractions, firſt of all reduce theſe Integers to the form of a Fraction by *N. B.* of *Rule* III. or reduce your Integer to a Fraction of the ſame *Denominator* with the given Fraction by *Rule* III. Thus, $8-\frac{2}{5}=\frac{8}{1}-\frac{2}{5}$ by *N. B. Rule* III. or $8-\frac{2}{5}=\frac{40}{5}-\frac{2}{5}$ by *Rule* III. In both caſes your Fractions are of the ſame kind, being ſimple Fractions.

Obſerve 2. If you have mixt numbers, reduce them to improper Fractions by *Rule* II. $6\frac{3}{4}+\frac{5}{8}=\frac{27}{4}+\frac{5}{8}$, both ſimple Fractions.

Obſerve 3. If you have compound Fractions, reduce them to ſimple ones by *Rule* IV.

$$\frac{8}{9}+\frac{3}{4}\text{ of }\frac{5}{6}=\frac{8}{9}+\frac{15}{24}.$$

Obſerve 4. If your Fractions are expreſſed in compoſite terms, you may reduce them to their loweſt terms by *Rule* VI.

The three firſt *Obſerves* are abſolutely neceſſary, the only uſe of this laſt is to make the operation more eaſy.

ADDITION *of* FRACTIONS.

REduce your Fractions to a common *Denominator*, (if they are not already ſo) and add the *Numerators* only, continuing your common *Denominator*.

EXAMPLES.

1. $\frac{8}{15}+\frac{3}{15}=\frac{11}{15}$

2. $\frac{3}{4}+\frac{5}{8}\,a=\frac{24}{32}+\frac{20}{32}=\frac{44}{32}\,b=1\,\frac{12}{32}\,c=1\,\frac{3}{8}$

Or $\frac{3}{4}+\frac{5}{8}\,f=\frac{6}{8}+\frac{5}{8}=\frac{11}{8}=1\,\frac{3}{8}$

3. $6\,\frac{2}{3}\,d+\frac{4}{5}=\frac{20}{3}+\frac{4}{5}\,a=\frac{100}{15}+\frac{12}{15}=\frac{112}{15}\,b=7\,\frac{7}{15}$

Or $6+\frac{2}{3}+\frac{4}{5}\,a=6+\frac{10}{15}+\frac{12}{15}=6+\frac{22}{15}\,b=6+1+\frac{7}{15}=7\,\frac{7}{15}$

4. $\frac{7}{9}+\frac{2}{3}$ of $\frac{4}{5}\,e=\frac{7}{9}+e\,\frac{8}{15}\,a=\frac{105}{135}+\frac{72}{135}=\frac{177}{135}\,b=1\,\frac{42}{135}$

5. $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}\,a=\frac{32}{64}+\frac{48}{64}+\frac{40}{64}=\frac{120}{64}\,b=1\,\frac{56}{64}\,c=1\,\frac{7}{8}$

Or $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}\,f=\frac{4}{8}+\frac{6}{8}+\frac{5}{8}=\frac{15}{8}=1\,\frac{7}{8}$

The citations of the *Rules* for working the above *Examples.*

a Rule VII.
b Rule I.
c Rule VI.
d Rule II.
e Rule IV.
f Cor. Rule VII.

Here, in *Example* I. my two Fractions, $\frac{8}{15}$ and $\frac{3}{15}$, having the ſame *Denominator* 15, I add the *Numerators* 8 and 3, which make 11; ſo the ſum of the two Fractions is $\frac{11}{15}$

In *Example* II. my two Fractions, $\frac{3}{4}$ and $\frac{5}{8}$, having different *Denominators*, *viz.* 4 and 8, I reduce the Fractions to a common *Denominator*, *viz.* 4×8=32, the common *Denominator*; then I multiply the *Numerator* 3, into 8, the *Denominator* of the other Fraction, *viz.* 3×8=24; ſo $\frac{24}{32}$ is the ſame with $\frac{3}{4}$: After the ſame manner I multiply the *Numerator* 5, into 4 the *Denominator*

minator of the other Fraction, *viz.* $5\times4=20$; ſo $\frac{20}{32}$ is the ſame with $\frac{5}{8}$; and inſtead of $\frac{3}{4}+\frac{5}{8}$, I have $\frac{24}{32}+\frac{20}{32}$, which Fractions having a common *Denominator* 32, I add the *Numerators* $24+20=44$; ſo the ſum of my two Fractions is $\frac{44}{32}$, which being an improper Fraction, I reduce to a mixt number, *viz.* I divide the *Numerator* 44, by the *Denominator* 32, the *Quot* comes out $1\frac{12}{32}$; and here the fractional part, $\frac{12}{32}$, being expreſſed in compoſite terms, I reduce it to its loweſt terms, *viz.* I divide both *Numerator* and *Denominator* by 4; ſo $\frac{12}{32}=\frac{2}{8}$, and the ſum of my two Fractions, *viz.* $\frac{3}{4}+\frac{5}{8}=1\frac{3}{8}$

This ſecond *Example* I work after another manner, thus, I find 4, the *Denominator* of the one Fraction $\frac{3}{4}$, meaſures 8, the *Denominator* of the other Fraction $\frac{5}{8}$; therefore (by *N. B. Rule* VII.) I multiply both *Numerator* and *Denominator* of the Fraction $\frac{3}{4}$ by 2, the number of times the *Denominator* 8 contains the *Denominator* 4, and I have $\frac{6}{8}$, a Fraction of the ſame value with $\frac{3}{4}$; ſo the two Fractions, $\frac{6}{8}$ and $\frac{5}{8}$, having the ſame common *Denominator* 8, I add the *Numerators* $6+5=11$; ſo the ſum of my two Fractions, *viz.* $\frac{3}{4}+\frac{5}{8}=\frac{11}{8}$; this, being an improper Fraction, I reduce it to a mixt number, and I have $\frac{3}{4}+\frac{5}{8}=\frac{6}{8}+\frac{5}{8}=\frac{11}{8}=1\frac{3}{8}$, as before.

In *Example* III. I firſt of all reduce my mixt number, $6\frac{2}{3}$, to an improper Fraction $\frac{20}{3}$, by *Rule* II. and then, inſtead of having a mixt number, $6\frac{2}{3}$, and a ſimple Fraction $\frac{4}{5}$ to add, I have two ſimple Fractions, *viz.* $\frac{20}{3}$, and $\frac{4}{5}$ to be added; ſo the caſe becomes the ſame as in *Example* II.

In *Example* IV. I firſt of all reduce my compound Fraction $\frac{2}{3}$ of $\frac{4}{5}$ to a ſimple Fraction $\frac{8}{15}$, and then, inſtead of a ſimple Fraction $\frac{7}{9}$ and a compound Fraction $\frac{2}{3}$ of $\frac{4}{5}$ I have two ſimple Fractions $\frac{7}{9}$ and $\frac{8}{15}$; ſo the caſe is again the ſame as in *Example* II.

In *Example* V. where I have three ſimple Fractions, $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}$, I reduce all the three to a common *Denominator* by *Rule* VII. *viz.* $2\times4\times8=64$, the common *Denominator*; then, $1\times4\times8=32$, the *Numerator* anſwering to 1, the *Numerator* of my firſt Fraction $\frac{1}{2}$, ſo $\frac{32}{64}=\frac{1}{2}$; then $3\times2\times8=48$, the *Numerator*, anſwering to 3, the *Numerator* of my ſecond Fraction $\frac{3}{4}$; ſo $\frac{48}{64}=\frac{3}{4}$; again $5\times2\times4=40$ the *Numerator* anſwering to 5 the *Numerator* of my third Fraction $\frac{5}{8}$, ſo $\frac{40}{64}=\frac{5}{8}$; then my three Fractions $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}=\frac{32}{64}+\frac{48}{64}+\frac{40}{64}$, and theſe laſt three Fractions having the ſame common *Denominator*, I add the *Numerators* thus, $32+48+40=120$; ſo the whole work ſtands thus,

$$\frac{1}{2}+\frac{3}{4}+\frac{5}{8}=\frac{32}{64}+\frac{48}{64}+\frac{40}{64}=\frac{120}{64}=1\frac{56}{64}=1\frac{7}{8}$$

As the two *Denominators*, 2 and 4, meaſure the *Denominator* 8, this *Example* may be wrought thus, 8 contains 2 four times, ſo $\frac{1}{2}=\frac{4}{8}$; again, 4 is contained in 8 twice; ſo $\frac{3}{4}=\frac{6}{8}$, and the whole ſtands thus,

$\frac{1}{2}+\frac{3}{4}+\frac{5}{8}=\frac{4}{8}+\frac{6}{8}+\frac{5}{8}=\frac{15}{8}=1\frac{7}{8}$, as before.

N. B. 1. Where the *Denominator* of one Fraction meaſures the *Denominator* of the other, you may either reduce your Fractions to a common *Denominator* by *Rule* VII. or, which is more compendious, by *N. B.* of *Rule* VII. as in *Examples* II. and V.

N. B. 2. In caſe of Mixt numbers, as in Caſe third, you may either reduce your mixt numbers to Improper Fractions, as in the firſt operation, or you may add the Simple Fractions only, and then add their ſum to the *Integer* in the Mixt number, as in the ſecond operation.

The reaſon of the *Rule* is plain; for, in order to the adding, or collecting things into one ſum, it is not ſufficient that they are of the ſame kind, but they muſt likewiſe be of the ſame name, or denomination; thus, tho' 7 and 8 make 15, yet 7 *Highlandmen* and 8 *Lowlandmen* neither make 15 *Highlandmen*, nor 15 *Lowlandmen*; but if they are reduced to the common denomination of *Scotſmen*, it is plain 7 *Scotſmen* and 8 *Scotſmen* make 15 *Scotſmen*; the ſame way, tho' 2 and 5 make 7, yet 2 thirds and 5 eights, neither make 7 thirds nor 7 eights: So they muſt be reduced to a common *Denominator* before they can be added.

SUBTRACTION *of* FRACTIONS.

REduce your Fractions to a common *Denominator*, (if they are not already ſo) and ſubtract the *Numerators* only, continuing your common *Denominator*.

EXAMPLES.

1. $\frac{8}{15} - \frac{3}{15} = \frac{5}{15}\,a = \frac{1}{3}$

2. $\frac{8}{9} - \frac{2}{3}\,b = \frac{24}{27} - \frac{18}{27} = \frac{6}{27}\,a = \frac{2}{9}$

Or $\frac{8}{9} - \frac{2}{3}\,c = \frac{8}{9} - \frac{6}{9}\,c = \frac{2}{9}$

3. $8\,d - \frac{2}{5} = \frac{8}{1}\,d - \frac{2}{5}\,b = \frac{40}{5} - \frac{2}{5} = \frac{38}{5}\,f = 7\frac{3}{5}$

Or $8\,e - \frac{2}{5} = \frac{40}{5}\,e - \frac{2}{5} = \frac{28}{5}\,f = 7\frac{3}{5}$

4. $4\frac{2}{3}\,g - \frac{3}{5} = \frac{14}{3}\,g - \frac{3}{5}\,b = \frac{70}{15} - \frac{9}{15} = \frac{61}{15}\,f = 4\frac{1}{15}$

5. $\frac{8}{9} - \frac{2}{3}\,k$ of $\frac{5}{6} = \frac{8}{9}\,k - k\,\frac{10}{18}\,b = \frac{144}{162} - \frac{90}{162} = \frac{54}{162}\,b = \frac{3}{9}\,b = \frac{1}{3}$

Or $\frac{8}{9} - \frac{2}{3}$ of $\frac{5}{6} = \frac{8}{9}\,c - \frac{10}{18} = \frac{16}{18} - \frac{10}{18} = \frac{5}{18} = \frac{1}{3}$

The citations of the *Rules* for working the above *Examples*.

a Rule VI.
b Rule VII.
c *N.B.* Rule VII.
d Cor. Rule III.
e Rule III.
f Rule I.
g Rule II.
k Rule IV.

N. B. from Caſe third. To ſubtract a Fraction from an Integer; Take 1 from the Integer, you have the Integer of the Remainder; ſubtract the *Numerator* of the Fraction from its *Denominator*, and you have the *Numerator* of the Fraction in the *Remainder*, whoſe *Denominator* is the ſame as before; thus $36 - \frac{6}{15} = 35\frac{9}{15} = 35\frac{3}{5}$

The reaſon of the *Rule* is the ſame as in *Addition*.

MULTIPLICATION *of* FRACTIONS.

MUltiply the *Numerators* for the *Numerator* of the *Product*, and the *Denominators* for its *Denominator*.

EXAMPLES.

1. $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$
2. $\frac{5}{6} \times 8 = \frac{5}{6} \times \frac{8}{1} = \frac{40}{6} = 6\frac{4}{6} = 6\frac{2}{3}$
3. $\frac{3}{4} \times 6\frac{1}{2} = \frac{3}{4} \times \frac{13}{2} = \frac{39}{8} = 4\frac{7}{8}$
4. $\frac{7}{9} \times \frac{2}{3}$ of $\frac{4}{5} = \frac{7}{9} \times \frac{8}{15} = \frac{56}{135}$

N. B. from Caſe ſecond. To multiply a Fraction by an Integer; Multiply the *Numerator* by the Integer, for the *Numerator* of the *Product*, and continue the *Denominator* the ſame.

The reaſon of the *Rule* is plain; for if $\frac{2}{3}$ and $\frac{4}{5}$ are to be multiplied, by the above *Theorem* $\frac{4}{5} = \frac{12}{15}$, and $\frac{1}{3}$ of $\frac{12}{15} = \frac{4}{15}$, and twice $\frac{4}{15} = \frac{8}{15}$; ſo $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$

COROLARY.

Hence Reduction of compound Fractions to ſimple Fractions, is Multiplication of the compound Fractions.

DIVISION *of* FRACTIONS.

MUltiply croſs-ways, *viz.* the *Numerator* of the *Dividend* by the *Denominator* of the *Diviſor*, for the *Numerator* of the *Quot*, &c.

EXAMPLES.

1. $\frac{2}{3})\,\frac{4}{5}\,(\frac{12}{10}=1\frac{2}{10}=1\frac{1}{5}$
2. $\frac{2}{3})\,8=\frac{2}{3}\,(\,\frac{8}{1}\,(\frac{24}{2}=12$
3. $6)\,\frac{4}{5}\,(=\frac{6}{1})\,\frac{4}{5}\,(\frac{4}{30}=\frac{2}{15}$
4. $\frac{3}{4})\,2\frac{2}{5}\,(=\frac{3}{4})\,\frac{12}{5}\,(\frac{48}{15}=3\frac{3}{15}=3\frac{1}{5}$
5. $\frac{5}{6})\,\frac{2}{3}$ of $\frac{4}{5}\,(=\frac{5}{6})\,\frac{8}{15}\,(\frac{48}{75}=\frac{16}{25}$

N. B. from Caſe ſecond. To divide an Integer by a Fraction; Multiply the Integer by the *Denominator*, for the *Numerator* of the *Quot*, and change the *Numerator* of the *Diviſor* to a *Denominator* in the *Quot*, $\frac{8}{9})15(\frac{135}{8}$

N. B. 2. from Caſe third. To divide a Fraction by an Integer; Multiply the *Denominator* by the Integer for the *Denominator* of the *Quot*, and continue the *Numerator* the ſame, $9)\,\frac{5}{8}\,(\frac{5}{72}$

The practice of this *Rule* is nothing elſe but reducing the Fractions to a common *Denominator*, and dividing the *Numerators*; thus, if $\frac{2}{3}$ of a Pound divide $\frac{4}{5}$ of a Pound, reducing them to a common *Denominator* $\frac{2}{3})\,\frac{4}{5}\,(=\frac{10}{15})\,\frac{12}{15}\,(=\frac{12}{10}=1\frac{2}{10}=1\frac{1}{5}$; for dividing 12 by 10. the *Quot* is $\frac{12}{10}$.

The *Examples* I have given of the Arithmetick of Fractions, are ſufficient to illuſtrate the whole doctrine; for, tho' I have only ſhown how to add or ſubtract a ſimple Fraction to, or from a mixt number; yet, as it appears in that *Example*, that the mixt number muſt firſt of all be reduced to an Improper Fraction, it will be eaſy to add or ſubtract a mixt number, to or from a mixt number, *viz.* by reducing both to Improper Fractions.

Having finiſhed the Arithmetick of Fractions, I ſhall here add one *Rule* more to the Reduction of Fractions, which may be of uſe in our following Doctrine.

RULE VIII. To reduce the known parts of any Integer to a Simple Fraction.

Reduce the known parts to the loweſt denomination, for the Numerator *of the Fraction; and the number of theſe parts contained in the Integer, is its* Denominator.

Thus 8 *s.* 6 *d.* = $\frac{102}{240}$ of *l.*

8 *s.*	1 *l.*
12	20
—	—
96	20
+6	12
—	—
102 *d.*	240 *d.*
—	—

Here 8 *s.* 6 *d.* = 102 *d.* and the Integer, or 1 *l.* = 240 *d.* ſo 8 *s.* 6 *d.* = $\frac{102}{240}$ The ſame way 5 months, 3 weeks, and 4 days = $\frac{165}{365}$ of a year.

5 *m.* 3 *w.* 4 *d.* = $\frac{165}{365}$ of a *Y.*

5 *m.*
4
—
20
3
—
23 *w.*
7
—
161
4
—
165
—

Nota, 365 Days make a Year, and is the *Denominator* of the Fraction.

RULES *for* PRACTICE.

RULES of PRACTICE naturally follow the doctrine of *Vulgar Fractions*, as being nothing elſe but a multiplying the ſum, or number of goods or commodities by ſuch a ſimple Fraction, or Fractions of a Pound, as is equivalent to the price of one of theſe commodities.

The aliquot parts of a Pound, and thoſe of a Shilling, with their equivalent ſimple Fractions of a Pound and of a Shilling, ſtand thus;

Parts of Pound.

s.	d.	
10	0	= $\frac{1}{2}$
6	8	= $\frac{1}{3}$
13	4	= $\frac{2}{3}$
5	0	= $\frac{1}{4}$
4	0	= $\frac{1}{5}$
3	4	= $\frac{1}{6}$
2	6	= $\frac{1}{8}$
2	0	= $\frac{1}{10}$
1	0	= $\frac{1}{20}$

Aliquot parts of a Shilling.

d.	
6	= $\frac{1}{2}$
4	= $\frac{1}{3}$
3	= $\frac{1}{4}$
2	= $\frac{1}{6}$
1$\frac{1}{2}$	= $\frac{1}{8}$
1	= $\frac{1}{12}$

If the *Numerator* of the Fraction is Unity, it is plain, that to multiply any number by that Fraction, is to divide the number by its *Denominator*; but if the *Numerator* is a number, firſt multiply the number of commodities by the *Numerator*, and then divide the *Product* by the *Denominator*; thus, if the price is 13 *s.* 4 *d.* = $\frac{2}{3}$, firſt multiply by 2, and then divide that *Product* by 3, or take $\frac{1}{3}$ of the number twice.

The whole doctrine will appear plainly in the following *Examples*.

EXAMPLE I.

What is the price of 87986 *lb.* of cotton-wooll, at 1 *s. per lb.*? *N. B.* 1 *s.* = $\frac{1}{20}$ *l.*

2|0)8798|6
4399 *l.* 6 *s.* *Anſwer.*

EXAMPLE II.

What is the price of 7648 yards, at 2 *s. per* yard? *N. B.* 2 *s.* = $\frac{1}{10}$ *l.*

1|0)764|8
764$\frac{8}{10}$ = 764 *l.* 16 *s.* *Anſ.*

EXAMPLE III.

What is the price of 7642 yards, at 2 *s.* 6 *d. per* yard? *N. B.* 2 *s.* 6 *d.* = $\frac{1}{8}$ *l.*

8)7642
955$\frac{2}{8}$ = 955 *l.* 5 *s.* *Anſ.*

EXAMPLE IV.

What is the price of 3468 *lb.* weight, at 3 *s.* 4 *d. per lb.*? *N. B.* 3 *s.* 4 *d.* = $\frac{1}{6}$ *l.*

6)3468
578 *l.* *Anſ.*

EXAMPLE V.

What is the price of 796 *lb.* weight, at 4 *s. per lb.*? *N. B.* 4 *s.* = $\frac{1}{5}$ *l.*

5)796
159$\frac{1}{5}$ = 159 *l.* 4 *s.* *Anſ.*

E X-

EXAMPLE VI.

What is the price of 868 *Scots* quarts of ſpirits, at 5 *s. per* quart? *N. B.* 5 *s.* = $\frac{1}{4}$ *l.*

4)868
217 *l. Anſ.*

EXAMPLE VII.

What is the price of 1768 yards, at 13 *s.* 4 *d. per* yard? *N. B.* 13 *s.* 4 *d.* = $\frac{2}{3}$ *l.*

1768
2

3)3536
1178 $\frac{2}{3}$ = 1178 *l.* 13 *s.* 4 *d. Anſ.*

This *Example* may be otherwiſe, and perhaps more expeditiouſly done by taking $\frac{1}{3}$ of the ſum twice, as in the margin.

3)1768

589 $\frac{1}{3}$
589 $\frac{1}{3}$

1178 $\frac{2}{3}$ *Anſwer*, as before.

EXAMPLE VIII.

What is the Price of 6548 *lb.* of coffee, at 6 *s.* 8 *d. per lb.*? *N. B.* 6 *s.* 8 *d* = $\frac{1}{3}$ *l.*

3)6548
2182 $\frac{2}{3}$ *l.* = 2182 *l.* 13 *s.* 4 *d. Anſ.*

EX-

EXAMPLE IX.

What is the price of 847 yards of holland, at 10 *s. per* yard? *N. B.* 10 *s.* = $\frac{1}{2}$ *l.*

2)847

423 $\frac{1}{2}$ = 423 *l.* 10 *s. Anſ.*

N. B. If the price of the Unit is any even number of Shillings, different from 2, 4, or 10, multiply $\frac{1}{2}$ of the price into the ſum given, always doubling the exceſs of the product of the right hand figure for Shillings, and carrying the tens into the product of the immediately following place for pounds.

EXAMPLE I.

What is the price of 4323 yards of cloth, at 6 *s. per* yard?

Here I multiply the yards 4323 by $\frac{1}{2}$ of the price, *viz.* 3 *s.* Thus, 3 times 3 make 9, which doubled, gives 18 Shillings; then 3 times 2 make 6; 3 times 3 make 9, and 3 times 4 make 12: So the *Anſwer* is 1296 *l.* 18 *s.*

432|3
3

1296 *l.* 18 *s.*

EXAMPLE II.

What is the price of 4678 yards, at 8 Shillings *per* yard?

Here I ſay, 4 times 8 is 32, and 2 (the exceſs above 3 tens) doubled, give 4 Shillings; then 4 times 7 is 28, and 3 tens that I carried make 31, I ſet down 1, and carry 3, &c.

467|8
4
————
1871 *l.* 4 *s.*

EXAMPLE III.

What is the price of 6478 yards of broad cloth, at 18 *s. per* yard?

647|8
9
————
5830 *l.* 4 *s.*

The reaſon of this practice is plain; for in *Example* I. 6 *s.* = $\frac{6}{20}$ *l.* = $\frac{3}{10}$; now the ſum multiplied by 6 *s.* and divided by 20, gives Pounds, therefore muliplying by 3, and dividing by 10, muſt likewiſe give Pounds; the cypher in the *Denominator* 10 cuts off the figure on the right, and makes it ten parts, but $\frac{1}{10}$ *l.* = 2 Shillings; therefore the *Product* of that figure, or its exceſs above ten or tens, muſt be multiplied by 2 to find the Shillings.

N.B. 2. If the price of the Unit of the wares is any odd number of Shillings, different from 5, work for the next leſſer even number of Shillings, as before, and for the odd Shilling take $\frac{1}{20}$ of the ſum given.

E X-

EXAMPLE.

What is the price of 859 yards, at 17 *s. per* yard?

85|9×8 = 687 *l.* 4 *s.* at 16 *s.*
85|9×$\frac{1}{20}$= 42 *l.* 19 *s.* at 1 *s.*

In all 730 *l.* 3 *s.* at 17 *s.*

The following *Examples* have the price of one of the commodities, the aliquot parts of a Shilling.

EXAMPLE I.

At 1 penny *per* Pound, what is the price of 9764 *lb.*? *N. B.* 1 *d.* = $\frac{1}{12}$ of a Shilling.

Here $\frac{1}{12}$ ×9764, gives 813 *s.* 8 *d.* and theſe Shillings reduced to Pounds, (dividing by 20) give 40 *l.* 13 *s.* So the full price is 40 *l.* 13 *s.* 8 *d.*

12) 9764 8 *d.*
2|0) 81|3 *s.*
40 *l.* 13 *s.*

EXAMPLE II.

What is the price of 7968 *lb.* at 1$\frac{1}{2}$ penny *per lb.*? *N. B.* 1$\frac{1}{2}$ *d.* = $\frac{1}{8}$ of a Shilling.

8)7968
2|0) 99|6 *s.*
49 *l.* 16 *s.* *Anſwer.*

EXAMPLE III.

What coſt 13147 *lb.* of raiſens, at 2 *d. per lb.* *N. B.* 2 *d.* = $\frac{1}{6}$ of a Shilling.

6)13147
2|0)219|1 0 2 *d.*
109 *l.* 11 *s.* 2 *d.* *Anſwer.*

EXAMPLE IV.

What coſt 87341 *lb.* of ſugar, at 3 *d. per lb.*? *N. B.* 3 *d.* = $\frac{1}{4}$ of a Shilling.

4)87341
2|0)2183|5 $\frac{1}{4}$ *s.* = 21835 *s.* 3 *d.*
1091 *l.* 15 *s.* 3 *d.* *Anſwer.*

EXAMPLE V.

What is the price of 3097 *lb.* of raiſins, at 4 *d. per lb.*? *N. B.* 4 *d.* = $\frac{1}{3}$ of a Shilling?

3)3097
2|0)103|2 $\frac{1}{3}$ *s.* = 1032 *s.* 4 *d.*
51 *l.* 12 *s.* 4 *d.* *Anſwer.*

EXAMPLE VI.

What coſt 78642 *lb.* of ſugar, at 6 *d. per lb.*? *N. B.* 6 *d.* = $\frac{1}{2}$ Shilling.

2)78642
2|0)39321
1966 *l.* 1 *s.* *Anſwer.*

Or, becauſe 6 *d.* = $\frac{1}{40}$ of a Pound, divide by 40, and the remainder is ſixpences; ſo the remainder 2 ſixpences make 1 Shilling.

4|0)7864|2
1966 *l.* 1 *s.* *Anſ.*

N. B. If the price of Unity, or one Integer of any commodity, is not an aliquot part of a Shilling, (as 5, 7, 8, 9, 10, or 11) you may either reſolve it into ſuch parts as are aliquot parts of a Shilling, or ſuch as are aliquot parts of a Pound, or of ſome juſt *Diviſor* of 20 Shillings, or a Pound.

EXAMPLE I.

What coſt 34071 *lb.* of figgs, at 5 *d. per lb.*?

Here 5 *d.*=3 *d.* + 2 *d.* = $\frac{1}{4}+\frac{1}{6}$ of a Shilling; ſo it is done, as in *Examples* IV. and III. immediately preceeding.

Or 5 *d.* = $\frac{1}{6}$ of half a crown.

And $\frac{1}{2}$ crown = $\frac{1}{8}$ of a Pound.

6)34071

8) 5678 $\frac{2}{6}$ *h. cr.*=5678 *h. cr.* 15 *d.* or 1 *s.* 3 *d.*

709 $\frac{6}{8}$ = 709 *l.* 16 *s.* 3 *d.*

EXAMPLE II.

What coſt 321 *lb.* of any thing, at 7 *d. per lb.*? *N. B.* 7 *d.* = 4 *d.* + 3 *d.* = $\frac{1}{3}+\frac{1}{4}$ of a Shilling.

Or 7 *d.*=4 *d.*+3 *d.*=$\frac{1}{60}+\frac{1}{80}$ of a Pound.

	l.	*s.*	*d.*
6|0)32|1=5$\frac{21}{60}$=	5	7	0
8|0)32|1=4$\frac{1}{80}$=	4	0	3

Anſwer - - - 9*l.* 7*s.* 3*d.*

EX-

EXAMPLE III.

What coſt 3746 yards of ribbon, at 8 *d. per* yard? *N. B.* 8 *d.* = $\frac{1}{30}$ of a Pound.

Here each Unit of the Remainder, 26, is 8 Pence.

3|0)374|6
124 *l.* 17 *s.* 4 *d.* *Anſwer.*

EXAMPLE IV.

What coſt 4052 *lb.* of any thing, at 9 *d. per lb.*? *N. B.* 9 *d.* = 6 *d.* + 3 *d.* = $\frac{1}{40}$ of a *lb.* + $\frac{1}{2}$ of $\frac{1}{40}$ of a Pound.

4|0)405|2
2) 101 *l.* 6 *s.*
50 13 *s.*

151 *l.* 19 *s.* *Anſwer.*

EXAMPLE V.

What coſt 3179 yards, at 10 *d. per* yard? *N. B.* 10 *d.* = 6 *d.* + 4 *d.* = $\frac{1}{40}$ + $\frac{1}{60}$ of a Pound.

L. *s.* *d.*
4|0)317|9 = 79 9 6
6|0)317|9 = 52 19 8

Anſwer 132 *l.* 9 *s.* 2 *d.*

If the price is 11 *d.* take $\frac{1}{12}$ of the ſum from it, and you have the price in Shillings.

EXAMPLE.

What coſt 7642 yards of cloth, at 11 *d. per* yard?

12)7642
636 *s.* 10 *d.*

2|0)700|5 2 *d.*
350 *l.* 5 *s.* 2 *d.* *Anſwer.*

If the price is Farthings under a Penny, find what Fraction it is of a Penny, or of a Shilling, and find the value of your goods in Pence or Shillings, which are eaſily reduced to Pounds.

EXAMPLE I.

What coſt 7258 yards of Tape, at 1 Farthing *per* yard?

Here the Farthing = $\frac{1}{4}$ of a Penny; ſo dividing 7258 yards by 4, gives Pence, and the Shillings and Pounds are eaſily found: So the *Anſwer* is 7 *l.* 11 *s.* 2 *d.* 2 *f.*

4)7258
12)1814 *d.* 2 *f.*
2|0) 15|1 2 *d.*
7 *l.* 11 *s.*

EX-

EXAMPLE II.

What coſt 38746 yards, at 2 *f.* *per* yard? 2 *f.* = $\frac{1}{2}$.

2)38746
12) 19373 *d.*
2|0) 161|4 *s.* 5 *d.*
80 *l.* 14 *s.*

Anſwer 80 *l.* 14 *s.* 5 *d.*

EXAMPLE III.

What coſt 56086 yards, at 3 *f.* *per* yard? 3 *f.* = $\frac{1}{2}$ *d.* + $\frac{1}{4}$ *d.*

2)56086 4)56086
$\frac{1}{2}$=28043 *d.* 14021 *d.* 2 *f.*
$\frac{1}{4}$=14021 *d.* 2 *f.*

12) 42064 *d.* 2 *f.*
2|0) 350|5 *s.* 4 *d.*
175 *l.* 5 *s.*

175 *l.* 5 *s.* 4 *d.* 2 *f.*

Or becauſe 3 halfpence = 2×3 Farthings, and 1 Shilling = 8×3 halfpence, take $\frac{1}{2}$ for 3 halfpence, and for Shillings $\frac{1}{8}$ of the 3 halfpence.

2)56086 3 *halfpence.*
8)28043 *s.*
2|0) 350|5 $\frac{3}{8}$ and $\frac{3}{8}$ = 4 *d.* 2 *f.*
175 $\frac{5}{20}$

Anſw. 175 *l.* 5 *s.* 4 *d.* 2 *f.*

Or becauſe 4×3 Farthings = 3 *d.* and 80×3 *d.* = 240 *d.* = 1 *l.* take $\frac{1}{80}$ of $\frac{1}{4}$ of the ſum, and you have Pounds. Here $\frac{1}{4}$ of 56086 = 14021 three Pences and $\frac{2}{4}$, or $\frac{1}{2}$ of 3 *d.* = 1 *d.* 2 Farthings remain.

Again, $\frac{1}{80}$ of 14021=175 and $\frac{21}{80}$, or 21 three Pences = 63 Pence remains, to which add the odd 1 Penny 2 Farthings, and you have of remainder 64 Pence 2 Farthings = 5 *s.* 4 *d.* 2 *f.* So the price is, as before, 175 *l.* 5 *s.* 4 *d.* 2 *f.*

4)56086 *l.* *s.* *d.* *f.*
8|0)1402|1 $\frac{1}{2}$ = 14021 0 1 2 = $\frac{1}{2}$
$\frac{1}{80}$ of $\frac{1}{4}$ = 175 $\frac{21}{80}$ = 175 5 4 2

If the price of the *Integer* is Shillings and Pence, and not an aliquot part of a Pound, divide the price into ſuch parts, as, at leaſt, one of theſe parts ſhall be an aliquot part of a Pound, work with that part, as directed in the Rules, for the aliquot parts of a Pound, and with the other part or parts, as directed in the Rules, for Shillings and Pence.

EXAMPLE I.

What coſt 796 yards of cloth, at 3 *s.* 10 *d.* *per* yard? 3 *s.* 10 *d.* = 3 *s.* 4 *d.* + 6 *d.*

3 *s.* 4 *d.* = $\frac{1}{6}$ of a Pound, and 6 *d.* = $\frac{1}{40}$ of a Pound.

796
$\frac{1}{6}$ = 132 *l.* 13 *s.* 4 *d.*
$\frac{1}{40}$ = 19 18 0

Makes 152 *l.* 11 *s.* 4 *d.*

EXAMPLE II.

At 17 *s.* 4 *d. per* yard, what coſt 394 yards? 17 *s.* 4 *d.* = 6 *s.* 8 *d.* + 6 *s.* 8 *d.* + 4 *s.* = $\frac{1}{3} + \frac{1}{3} + \frac{1}{5}$

394				
131 *l.*	6 *s.*	8 *d.*	=	$\frac{1}{3}$
131	6	8	=	$\frac{1}{3}$
78	16	0	=	$\frac{1}{5}$
341 *l.*	9 *s.*	4 *d.*	*Anſwer.*	

EXAMPLE III.

What coſt 1504 yards of cloth, at 19 *s.* 9 *d. per* yard?

Here, at 18 *s.* I multiply by 9, doubling the Units of the product for Shillings, as before taught; then at 1 *s.* I take $\frac{1}{20}$, at 6 *d.* I take $\frac{1}{40}$, and $\frac{1}{2}$ of that at 3 *d.* Theſe added give the *Anſwer.*

1504				
1353 *l.*	12 *s.*	0 *d.*	0 *f.*	at 18 *s.*
75	4	0	0	at 1 *s.*
37	12	0	0	at 6 *d.*
18	16	0	0	at 3 *d.*
1485 *l.*	04 *s.*	0 *d.*	0 *f.*	*Anſw.*

EX-

EXAMPLE IV.

At 14 *s.* 10 *d.* what coſt 1904 yards?

Here I multiply by half my Shillings, *viz.* 7, as before, and 10 *d.* = 6 *d.* + 4 *d.* = $\frac{1}{40} + \frac{1}{60}$

	1904				
	1332	16 *s.*	0 *d.*	0 *f.*	at 14 *s.*
$\frac{1}{40}$	47	12	0	0	at 6 *d.*
$\frac{1}{60}$	31	14	8	0	at 4 *d.*
	1412 *l.*	02 *s.*	8 *d.*	0 *f.*	*Anſwer.*

EXAMPLE IV.

What coſt 1865 yards, at 2 *s.* 4 *d. per* yard?

Here, inſtead of taking $\frac{1}{60}$ of the number of yards for 4 *d.* I might have taken $\frac{1}{6}$ of the price at 2 *s.* for 4 *d.* = $\frac{1}{6}$ of 2 Shillings.

	1865				
	186 *l.*	10 *s.*	0 *d.*	0 *f.*	at 2 *s.*
$\frac{1}{60}$ =	31	1	8	0	at 4 *d.*
	217 *l.*	11 *s.*	8 *d.*	0 *f.*	*Anſwer.*

If the price is Pence and Farthings, work for Pence as before; if the Farthings are an aliquot part of the Pence, take that part of the price at Pence, otherwiſe work for your Farthings, as before taught, where the price was only Farthings.

EX-

EXAMPLE I.

What coſt 3471 dozen of buttons, at 3 *d.* $\frac{1}{2}$ *per* dozen?

Here 3 *d.* = $\frac{1}{80}$ of a Pound, and $\frac{1}{2}$ *d.* = $\frac{1}{6}$ of 3 *d.* So I firſt take $\frac{1}{80}$ of the number of dozens, and then $\frac{1}{6}$ of that.

	3471				
$\frac{1}{80}$ =	43 *l.*	7 *s.*	9 *d.*	0 *f.*	at 3 *d.*
$\frac{1}{6}$ of the above	7	4	7	2	at $\frac{1}{2}$ *d.*
	50 *l.*	12 *s.*	4 *d.*	2 *f.*	*Anſ.*

EXAMPLE II.

What coſt 9761 *lb.* of ſugar, at 5 *d.* 1 *f. per* Pound?

Here 5 *d.* = 3 *d.* + 2 *d.* = $\frac{1}{80}$ + $\frac{1}{120}$, and 1 Farthing = $\frac{1}{8}$ of $\frac{1}{120}$, or 2 *d.*

	9761				
$\frac{1}{80}$ =	122 *l.*	0 *s.*	3 *d.*	0 *f.*	at 3 *d.*
$\frac{1}{120}$ =	81	6	10	0	at 2 *d.*
$\frac{1}{8}$ of $\frac{1}{120}$ =	10	3	4	1	at 1 *f.*
	213 *l.*	10 *s.*	5 *d.*	1 *f.*	*Anſw.*

EXAMPLE III.

What coſt 1794 *lb.* of pepper, at $3\frac{3}{4}$ *d. per lb.?*

Here 3 *d.* = $\frac{1}{80}$ of a Pound, and 3 *f.* = $\frac{1}{4}$ of $\frac{1}{80}$, or $\frac{1}{4}$ of 3 *d.*

1794

$\frac{1}{80}$ = 22 *l.* 8 *s.* 6 *d.* 0 *f.* at 3 *d.*
$\frac{1}{4}$ of $\frac{1}{80}$ = 5 12 1 2 at 3 *f.*

28 *l.* 00 *s.* 7 *d.* 2 *f.* *Anſwer.*

EXAMPLE IV.

What coſt 3475 yards of Ribbon, at 4 *d.* $\frac{2}{3}$ *per* yard?

Here 4 *d.* = $\frac{1}{60}$ of a Pound, and $\frac{2}{3}$ = $\frac{1}{6}$ of 4 *d.* So I firſt take $\frac{1}{60}$ of the number of yards, and then $\frac{1}{6}$ of that.

3475

$\frac{1}{60}$ = 57 *l.* 18 *s.* 4 *d.* at 4 *d.*
$\frac{1}{6}$ of the above 9 13 $00\frac{2}{3}$ at $\frac{2}{3}$

67 *l.* 11 *s.* $4\frac{2}{3}$ *d.* *Anſwer.*

If the price is Pounds, Shillings, Pence and Farthings, multiply the number given by the Pounds, and work for the Shillings, Pence and Farthings, as before.

EX-

EXAMPLE.

What coſt 276 hundred weight 2 quarters of ſteel, at 2 *l.* 3 *s.* 8½ *d. per* hundred?

	276				
	2				
	552 *l.*	0 *s.*	0 *d.*	0 *f.*	at 2 *l.*
$\frac{1}{6}$ =	46	0	0	0	at 3 *s.* 4 *d.*
$\frac{1}{60}$ =	4	12	0	0	at 4 *d.*
$\frac{1}{8}$ of $\frac{1}{60}$ =	0	11	6	0	at 2 *f.*
	603	03	6	0	= 276 *hundred.*
	1	1	10	1	= *half hundred.*
	604 *l.*	5 *s.*	4 *d.*	1 *f.*	*in all.*

Here I firſt find the price of 276 hundred weight, *viz.* for Pounds I multiply by 2 *l.* and work for the reſt, as before directed; then to the price of 276 hundred, thus found, I add half the price of one hundred for the odd half hundred weight, and the full price is 604 *l.* 5 *s.* 4 *d.* 1 *f.*

DECIMAL FRACTIONS.

DECIMAL FRACTIONS are ſuch as have 10, 100, 1000, 10000, or Unity with any number of cyphers annexed to it for a *Denominator*.

Thus $\frac{6}{10}$ $\frac{25}{100}$ $\frac{785}{1000}$ $\frac{8}{10000}$ are *Decimal Fractions.*

Theſe Fractions may be otherways written thus, .6, .25, .785, .0008, where the point on the left hand, called, *The Decimal Point*, repreſents Unity (the only ſignificant figure) in the *Denominator*, and the number of figures on the right hand of that point, called, *The number of Decimal Places*, ſhews the number of cyphers belonging to the *Denominator*; thus, .6 has but one decimal place, ſignifying one cypher only to belong to the *Denominator*; .25 has two decimal places, ſignifying two cyphers belong to the *Denominator*; and .785 has three decimal places, ſignifying three cyphers belong to the *Denominator*, &c.

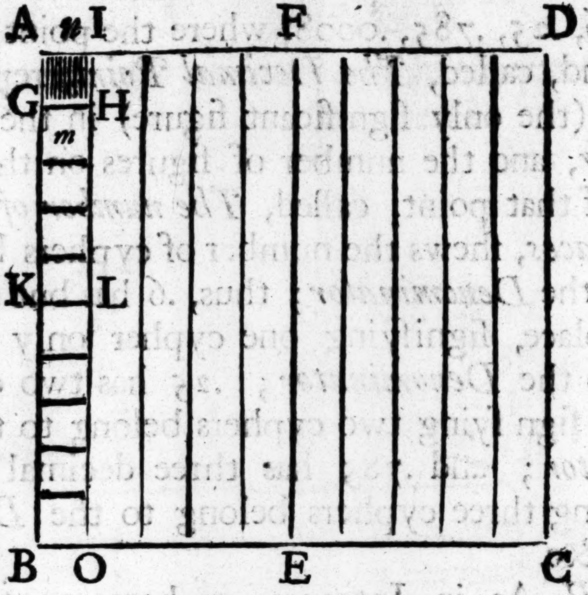
N. B. As in Integers, cyphers on the left hand of ſignificant figures do not change their value, ſo in *Decimals*, cyphers on the right hand of ſignificant figures do not change the value of the *Decimal*.

Thus, as in Integers, 05 $=$ 5, and 005 $=$ 5; ſo in Decimals .50 $=$ .5, and .500 $=$ .5; for .50 $= \frac{5|0}{10|0} = \frac{5}{10} =$ .5, and .500 $= \frac{5|00}{10|00} = \frac{5}{10}$ $=$ .5

N. B. 2. As in Integers, cyphers on the right hand of ſignificant figures augment their value in a decuple proportion; ſo in Decimals cyphers on the left hand of ſignificant figures, diminiſh or leſſen their value in a decuple proportion.

Thus, as in Integers, 5 $=$ five, 50 $=$ 5 tens, or fifty, 500 $=$ five hundred; ſo in Decimals .5 $= \frac{5}{10}$, .05 $= \frac{5}{100}$, .005 $= \frac{5}{1000}$

Thus



Thus making ABCD the Integer, and dividing it into 10 equal parts by lines IO, FE, &c. any one of theſe parts is $\frac{1}{10}$ of ABCD, and 5 of theſe parts, as ABEF = .5 of ABCD and dividing any one of theſe 10th parts, as ABOI, into 10 equal parts, any one of theſe parts, as AGHI, is $\frac{1}{10}$ of ABOI, and conſequently AGHI = $\frac{1}{100}$ of ABCD, and 5 of theſe parts AKLI = .05 of ABCD. Again, dividing one of theſe 100th parts, as AGHI, into 10 equal parts, any one of theſe is $\frac{1}{10}$ of AGHI; (which is $\frac{1}{100}$ of ABCD) it is therefore $\frac{1}{1000}$ of ABCD, and 5 of theſe parts, as AG*mn* = .005 of ABCD, which is ten times leſs than the former *Decimal*, *viz.* AKLI = .5

Reduction of DECIMALS.

RULE I. To reduce *vulgar Fractions* to *Decimals.*

Divide the Numerator *by the* Denominator, (*annexing cyphers to the* Numerator, *till it is a ſufficient* Dividend) *the* Quot *is the* Numerator *of the* Decimal, *and the number of cyphers annexed gives the number of* decimal *places.* Thus $\frac{3}{4} = .75$

EXAMPLE I.

Here, to the *Numerator* 3, I annex one cypher, which makes 30, I have 7 in the *Quot*, and 2 remain, to which I annex another cypher, which makes 20, I divide, and have 5 in the *Quot*, and 0 remains: So 75 is the *Numerator* of my *Decimal*, and for the two cyphers annexed I allow two *decimal* places. So $\frac{3}{4} = .75$

```
4)30(75
  28
  ——.
   20
   20
   ——
    0
```

The *Rule* is the ſame with the *Problem* in *vulgar Fractions*; (*page* 107) for annexing the two cyphers is the ſame as if I had propoſed to reduce $\frac{3}{4}$ to a Fraction, whoſe *Denominator* is 100.

```
   3
  100
  ———
4)30,0,(75
  28
  ———
   20
   20
   ——
    0
```

N. B. If, in reducing a *vulgar Fraction* to a *Decimal*, o remains at laſt, your *Decimal* is limited, as being preciſely equal to the *vulgar Fraction*, and is called a *finite Decimal.*

EXAMPLE. II.

$\frac{3}{848} = .0035377$

N. B. 2. If you have ſtill a remainder, you may limit your *Decimal* at three figures in the *Quot*, or four or five, *&c.* but never under three: The reaſon of thus limiting is, three figures in the *Decimal* divides the Integer into 1000 equal parts, and all you loſe by limiting at three places, is a Fraction of $\frac{1}{1000}$ of the Integer, and all you loſe by limiting at four places, is a Fraction of $\frac{1}{10000}$ of the Integer, *&c.*

```
           ...
848)3000(35377
    2544
    ----.
    4560
    4240
    ----.
     3200
     2544
     ----.
     6560
     5936
     ----.
      6240
      5936
      ----
       304
      ----
```

N. B. 3. If your *Quot* has not ſo many places (or figures) as you have annexed cyphers, ſupply theſe deficient places with cyphers on the left hand of the *Quot*, before you ſet your *decimal* point, as in *Example* II.

EX-

EXAMPLE III.

$\frac{14}{15} = .93\cancel{3}$

N. B. 4. If (as in *Example* III.) your *Remainder* is ſtill the ſame, and conſequently the ſame figure is continually repeted in the *Quot*, your *Decimal* is infinite, and is called an *infinite repeting Decimal.*

```
15)140(93̸3̸
   135
   ——.
    50
    45
    ——.
    50
    45
    ——.
    50
    45
    ——
     5
    ——
```

Theſe *Decimals* are, for ordinary, marked with a daſh through the right hand figure.

The ſame way $\frac{1}{9} = 11\cancel{1}$ an *infinite repeting Decimal.*

```
9)10(111̸
  9
  —.
  10
   9
  —.
  10
   9
  —
   1
  —
```

EXAMPLE IV.

$\frac{9}{28} = .3214285714$, &c.

N. B. 5. If, in your *Quot*, you have a continued revolution of a certain number of figures always the ſame, (as in *Example* IV.) you have 142857, again, 142857, and ſo on infinitely, your *Decimal* is infinite, and is called an *infinite circulating Decimal*, or, *A Circulate*.

```
       .
28)90(3214285714
   84
   —.
   60
   56
   —.
     40
     28
     —.
     120
     112
     ——.
       80
       56
       —.
       240
       224
       ——.
        160
        140
        ——.
         200
         196
         ——.
          40
          28
          —.
          120
          112
          ——
            8&c.
```

The ſame way $\frac{3}{11}$ = .2727272 is an *infinite circulating Decimal*, having a continued revolution of 27 and 27 and 27, *in infinitum*.

11)30(2727272
22
—.
80
77
—.
30
22
—.
80
77
—.
30
22
—.
80
77
—.
30
22
—
8

For further illuſtration of this, I ſhall add a few more *Examples*; as,

1. What is the *Decimal* of 16 *s.* 6 *d.* a Pound being the *Integer* ?

16 *s.* 6 *d.* = $\frac{198}{240}$ = .825

The *Decimal* is finite.

```
                  .
 16     240)1980(825
 12         1920
 —          ————.
 32          600
16           480
—            ——.
192         1200
+6          1200
—           ————
198            0
```

2. What is the *Decimal* of 18 *s.* 4 *d.* the Pound being the *Integer* ?

18 *s.* 4 *d.* = $\frac{220}{240}$ = .9166

The *Decimal* is infinite, repeting 666; for the remainder, 160, is always the ſame: Therefore the figure in the *Quot* is ſtill the ſame, 6.

```
                    .
 18     240)2200(9166
 12         2160
 —          ————.
 36           400
18            240
—-            —-.
216          1600
+4           1440
—.           ——.
220          1600
             1440
             ————
              160
```

3. What is the *Decimal* of .15 *lb.* weight, a quarter hundred being the *Integer?*

N.B. The great *Ct.* being 112 *lb.* weight, the quarter is 28 *lb.* weight; ſo 15 *lb.* weight = $\frac{15}{28}$ = .53571428571428,&c *in infinitum*, an infinite circulating *Decimal*, the circulating figures being 571428.

28)150(5357142857
140
—.
100
84
—.
160
140
—.
200
196
—.
40
28
—.
120
112
—.
80
56
—.
240
224
—.
160
140
—.
200
196
—
4, &c.

RULE II. To continue any Diviſion to a *Decimal.*

> *After your* Dividend *is exhauſted, ſet a point on the right hand of the* Quot, *annex cyphers to the right hand of the* Remainder, *and continue your* Diviſion *as before. The number on the right hand of the point is a* Decimal Fraction *of the* Integer, *expreſſed by the* Quot.

Thus, dividing 79689 *l.* amongſt 24 men, each man has 3320 *l.* and .375 of a Pound, or (as will appear afterward) 3320 *l.* 7 *s.* 6 *d.*

```
24)79689(3320.375
   72
   ——
    76
    72
    ——
     48
     48
     ——.
      090
       72
       ——.
       180
       168
       ——.
        120
        120
        ———
          0
```

RULE III. To value *Decimals* in the known parts of the *Integer*.

Multiply the Decimal *by the number of known parts contained in the* Integer, *and from the* Product *cut off as many* decimal *places as are in the* Decimal *given; ſo you have the parts on the left hand of the point, and on the right a* Decimal *of one of theſe parts: So this* Decimal *of a Pound* .876 = 17 *s.* 6 *d.* 0.96 *f.*

Here I multiply .876 *l.* by 20 Shillings in the Pound, and from the *Product* I cut off three *decimal* places, (anſwering to the three places in .876) and I have 17 *s.* and .520, or .52 of a Shilling, for the cypher on the right of .52 does not change its value; theſe .52 I multiply by 12 Pence in a Shilling, and I have 6.24, or 6 Pence and .24 of a Penny, which I multiply by 4 Farthings in a Penny, and find the *Product* .96 of a Farthing.

```
   .876  l.
     20
 ———————
 17.520  s.
     12
   ———
   104
   52
  ————
   6.24  d.
      4
     —
   0.96  f.
```

This *Decimal* of an acre .746 = 2 *roods*, 39 *falls*, 12.96 *ells*.

Here I multiply .746 of an acre by 4 roods in an acre, and find 2 roods and .984 of a rood, which I multiply by 40 falls in a rood, and find 39 falls and .360, or .36 of a fall, which, multiplied by 36 ells in a fall, gives 12 ells, and .96 of an ell.

```
   .746  a.
      4
  -----
  2.984  r.
     40
 ------
 39.360  f.
     36
 ------
    216
   108
 ------
  12.96  e.
 ======
```

RULE IV. To value the *Decimals* of a Pound by inſpection.

Double the figure on the left hand, (i. e. *next to the point*) *gives Shillings; divide the two following figures* (*if leſs than* 25) *by* 4, *the* Quot *gives Pence, the* Remainder *Farthings: But if the two following figures are* 25, *or more, deduce one before you divide by* 4.

EXAMPLES.

	L.	s.	d.	f.
So	.718 =	14	4	2
	.625 =	12	6	0
	.843 =	16	10	2
	.509 =	10	2	1
	.036 =	0	8	3
	.92 =	18	5	0
	.7 =	14	0	0

In *Example* 1. double the right hand figure 7, gives 14 *s.* I divide the two following figures 18 (being leſs than 25) by 4, the *Quot* gives 4 *d.* and the *Remainder* 2 Farthings.

In *Example* 2. the two laſt figures being 25, I deduce 1 before I divide by 4, ſo 24 divided by 4 gives 6 *d.*

In *Example* 6. having only two figures in the *Decimal* .92, I conceive a cypher on the right hand, which makes it .920.

N. B. If the figure in the 2d place from the point is 5, or more than 5, add 1 to your Shillings; reject 5 in the 2d place, and conceive the exceſs above 5 in the 2d place as tens before the figure in the 3d place, and then find your Pence and Farthings, as before.

EXAMPLES.

L.	*s.*	*d.*	*f.*
.759 =	15	2	1
.875 =	17	6	0
.95 =	19	0	0
.68 =	13	7	1
.072 =	1	5	2

In *Example* 1. twice 7 is 14 *s.* to which adding 1, (becauſe the figure in the 2d place is 5) I have 15 *s.* and rejecting the 5, there reſts 9, which gives 2 *d.* 1 *f.*

In *Example* 2. twice 8 gives 16 *s.* to which adding 1 *s.* (becauſe 7 in the 2d place is more than 5) we have 17 *s.* reject 5 in the 2d place and 2 remain, which placed as 2 tens before 5, (the figure in the 3d place) makes 25, which affords 6 *d.*

The reaſon is plain; for the Pound containing 20 *s.* 'tis certain .1 of a Pound = 2 *s.* ſo double the figure next the point, gives Shillings; and, as .1 = 2 Shillings, its half .05 = 1 Shillings, ſo 5 in the 2d place adds 1 Shilling to thoſe found, by doubling the figure next the point.

Again, The Pound contains 960 Farthings, and a *Decimal* of 3 places divides the Pound into 1000 equal parts, which makes an error of 40, and if 1000 errs in 40, its half, 500, errs in 20, and $\frac{1}{10}$ of 500, or 50, errs in 2, and its half, 25, errs in 1 Farthing; when therefore the two figures in the 2d and 3d places are 25, or more, I deduce the 1 of error ariſing from the diviſion of the Pound into 1000, inſtead of 960 Farthings.

N. B. If in 25 there is an error of 1, in 50 there muſt be an error of 2, and in 75 an error of 3 Farthings.

RULE V. To reduce Shillings, Pence and Farthings to *Decimals*, by inſpection; the *Decimal* being only 3 figure deep.

> *Half your Shillings gives your figure on the left hand, or next the point. Multiply your Pence by* 4. *to the* Product, (*if it is leſs than* 24) *add your Farthings for the two following Figures: But if the* Product *is* 24, *or more, add to it* 1 *before you add your Farthings.*

We may account for adding 1 to the *Product* of our Pence multiplied by 4 when the *Product* is 24, or more, and adding 2 to the *Product* when it is 48, or more, &c. Thus, dividing 960 by 24, the *Quot* is 40, the difference betwixt 960 and 1000; if, therefore, for every 24 in 960 we add 1, we change the 960 parts to 1000 parts, or we change the *Denominator* from 960 to 1000. Thus $\frac{24}{960} = \frac{24+1}{1000} = \frac{25}{1000}$ and $\frac{48}{960} = \frac{48+2}{1000} = \frac{50}{1000}$ and $\frac{72}{960} = \frac{72+3}{1000} = \frac{75}{1000}$; but $\frac{25}{1000} = .025$, $\frac{50}{1000} = .050 = .05$ and $\frac{75}{1000} = .075$

N. B. If the number of Shillings is odd, take half of the next leſſer even number for your left hand figure, and add .050 to the *Decimal* found, as before. The *Rule* is the reverſe of the preceeding.

EXAMPLES.

s.	*d.*	*f.*	
16	4	2	=.818
12	8	3	=.636
6	2	0	=.308
8	3	1	=.413
2	6	0	=.125
0	1	1	=.005

In *Example* 1. half my 16 Shillings gives 8, for the figure next the point. My 4 *d.* I multiply by 4, and have *Product* 16, to which (as being leſs than 24) I add my 2 Farthings, which makes the *Decimal* .818 = 16 *s.* 4 *d.* 2 *f.*

In *Example* 2. $\frac{1}{2}$ my Shillings gives 6 on the left hand; I multiply my 8 *d.* by 4, to the *Product* (as being more than 24) I add 1 and my 3 Farthings; ſo I have a *Decimal* .636 = 12 *s.* 8 *d.* 3 *f.*

EXAMPLES.

s.	*d.*	*f.*	
15	2	1	=.759
17	6	2	=.877
13	4	0	=.666
19	0	0	=.95
1	3	0	=.062

In *Example* 1. I ſet 7 next the point, as being half 14, the even number next to, and leſs than 15: I multiply my 2 *d.* by 4, which gives 8, and to it (becauſe it is leſs than 24) I add my 1 *f.* which make 9, to which (becauſe of the odd Shilling) I add 50; ſo I have .759 = 15 *s.* 2 *d.* 1 *f.*

In *Example* 2. I ſet 8 next the point; I multiply my 6 *d.* by 4; to the *Product* 24 I add 1, and my 2 *f.* which make 27, to which (becauſe of the odd Shilling) I add .050; ſo I have .877 = 17 *s.* 6 *d.* 2 *f.*

In *Example* 4. I ſet 9 (*viz.* half 18) next the point, to which I annex .050 becauſe of the odd Shilling; but, becauſe cyphers on the right hand of *Decimals* do not change their value, I neglect the cypher on the right hand of .050; ſo I have .95 = 19 *s.*

Addition and Subtraction of DECIMALS.

WORK as in *Integers*, keeping all your *Decimal* points directly below each other.

Examp. of Addition.		*Examp. of Subtraction.*	
.75	86.5	.75	82.125
.895	79.725	.275	26.75
.5	18.75		
.625	24.5	.475	55.375
.78	96.865		
.125			
	306.34		
3.675			

The *Rule* is the ſame as in *Addition of vulgar Fractions*; for, conſidering that cyphers on the right hand of a *Decimal* do not change its value, the firſt *Example* of *Addition* will ſtand thus.

Here the *Decimals* are all at a common *Denominator*, *viz.* 1000.

.750 = .75
.895 = .895
.500 = .5
.625 = .625
.780 = .78
.125 = .125

3.675 = 3.675

And the firſt *Example* of *Subtraction* will ſtand thus.

Here the *Decimals* are at a common *Denominator*, *viz.* 1000.

$$\begin{array}{r@{\;=\;}r} .750 & .75 \\ .275 & .275 \\ \hline .475 & .475 \\ \hline \end{array}$$

Multiplication of DECIMALS.

WORK as in *Integers*, and give as many *Decimal* places to the *Product* as are in both *Factors*.

Example I.	*Example* II.	*Example* III.
.785	76.85	.125
.75	9.5	.25
3925	38425	625
5495	69165	250
.58875	730.075	.03125

The *Rule* is the ſame as in *Multiplication* of *vulgar Fractions*; for $.75 \times .785 = \frac{75}{100} \times \frac{785}{1000} = \frac{58875}{100000} = .58875$

N. B. from *Example* III. If your *Product* has not ſo many figures as there are *Decimal* places in both *Multiplicand* and *Multiplier*, ſupply theſe places with cyphers on the left hand of the *Product* before you ſet your *Decimal* point.

Diviſion of DECIMALS.

WORK as in *Integers*, and give as many *Decimal* places to the *Quot* as the *Dividend* has more than the *Diviſor*.

EXAMPLE I.

In this *Example* the *Dividend* has five *Decimal* places, and the *Diviſor* two; ſo the *Dividend* has three places more than the *Diviſor*, which determines the number of *Decimal* places due to the *Quot* to be three.

3.75)87.94345(23.451
750''''
1294
1125
1693
1500
1934
1875
595
375
220

EXAMPLE II.

In this ſecond *Example* the *Dividend* has two *Decimal* places, the *Diviſor* two, the *Dividend* has no more *decimal* places than the *Diviſor*; ſo there is no *decimal* place due to the *Quot*. Hence,

3.25)76.75(23
650'
1175
975
200

If the *Diviſor* and *Dividend* have an equal number of *decimal* places, the *Quot* comes out an *Integer*.

Example II. *continued to a* Decimal.

N. B. In this and the like caſes, where the *Quot* is an *Integer*, or has not a ſufficient number of *decimal* places, it may be continued to any number of places, by annexing cyphers to the *Remainder*, after the *Dividend* is exhauſted by *Rule* II. of *Reduction of* Decimals: Thus, in dividing 76.75 by 3.25, after the *Dividend* was exhauſted, I annexed cyphers till I brought my *Quot* (23.615) to a *Decimal* of three places, and as there is ſtill a *Remainder*, I can carry it further, but three ſuffices.

```
3.25)76.75(23.615
     650'
     ——
     1175
      975
     ——.
      2000
      1950
      ———.
        500
        325
        ——.
        1750
        1625
        ————
         125
        ————
```

EXAMPLE III.

N. B. 2. If your *Diviſor* has *decimal* places, and your *Dividend* none, or if your *Diviſor* has more *decimal* places than your *Dividend*, ſupply the deficient places in your *Dividend* with cyphers, and the *Quot* is an *Integer*, as in *Examples* III. and IV.

```
3.25)7968(
3.25)7968.00(2451
     650' ' '
     ----
     1468
     1300
     ----
      1680
      1625
      ----
        550
        325
        ---
        225
```

EXAMPLE IV.

It is plain, that, as there are *Remainders* in theſe *Examples*, they may be continued to a *Decimal*, as in *Example* II.

```
2.375)7658.5(
2.375)7658.500(3224
      7125' ' '
      ----
      5335
      4750
      ----
       5850
       4750
       -----
       11000
        9500
       -----
        1500
       -----
```

Obſerve 1. If your *Diviſor* is 10, 100, 1000, *&c.* and your *Dividend* an *Integer*, cut off from the *Dividend* as many *decimal* places as there are cyphers in the *Diviſor*, and you have the *Quot*. Thus 875 *l.* divided amongſt 10 men, give 87.5 =87 *l.* 10 *s.*

	l.	*l.*	*s.*	*d.*	*f.*
10)875(	87.5 =	87	10	0	0
100)875(	8.75 =	8	15	0	0
1000)875(	.875 =	0	17	6	0
10000)875(	.0875 =	0	1	9	0

Obſerve 2. If your *Diviſor* is 10, 100, 1000, *&c.* and your *Dividend* has *decimal* places, augment the number of *decimal* places in the *Dividend* by the number of cyphers in the *Diviſor*, and you have the *Quot*. Thus, if 28 *l.* 15 *s.* = 28.75 are to be divided amongſt 10 men, each man has 2.875 = 2 *l.* 17 *s.* 6 *d.*

	l.	*l.*	*s.*	*d.*	*f.*
10)28.75(	2.875 =	2	17	6	0
100)28.75(	.2875 =	0	5	9	0
1000)28.75(	.02875 =	0	0	6	3

The reaſon of giving as many *decimal* places to the *Quot*, as the *Dividend* has more than the *Diviſor*, is, *Multiplication* gives as many *decimal* places to the *Product*, as are in both *Factors*; the *Dividend*, being nothing elſe but the *Product* of the *Diviſor* and *Quot*, muſt have as many *decimal* places as are in both, conſequently the *Quot* muſt have as many *decimal* places as the *Dividend* has more than the *Diviſor*.

N. B. I here, with moſt Writers on this ſubject, call the figure in the *Decimal*, which ſtands next to the point, or on the right hand of the point, *Decimal Primes*, the ſecond figure from the point I call *Decimal Seconds*, the next, *Thirds*, *&c.* Thus, in this *Decimal* .875, the figure next the point, (*viz.* 8.) is *decimal Primes*, 7 is *Seconds*, 5 *Thirds*.

We ſhall here give another *Rule* for placing the *decimal* point in the *Quot*, ariſing from the nature of *Diviſion*, and founded on *N. B.* *Page* 29. where we obſerved, That the firſt *Dividual*, with its ſubſequent figures, gives the number of figures in the *Quot*, or, which is the ſame thing, whatever is the place, in the order of *Numeration*, of the figure in the *Dividend* ſtanding above the Units of the *Product* of the *Diviſor*, and firſt figure of the *Quot*, the ſame place, in the order of *Numeration*, the firſt figure of the *Quot* obtains. This is the univerſal *Rule* in diviſion of *Integers*.

Thus, dividing 27685 by 35, I find that 6, the figure of my *Dividend*, ſtanding above 5, the Units of my firſt *Product*, is in the place of hundreds in my *Dividend*; therefore 7, the firſt figure of my *Quot*, is likewiſe hundreds; ſo the *Quot* is 791 *Integers*.

```
35)27685(791
   245··
   ———
    318
    315
    ———
      35
      35
      —
       0
```

But, if the *Dividend* 2768.5 has one *decimal* place (.5) the Units of my firſt *Product*, ſtands under tens of the Integers of my *Dividend*; ſo 7, the firſt figure of my *Quot*, is tens, 9, the next figure, is Units, and the following figure .1 is a *decimal Prime*.

35)2768.5(79.1
245 · ·
318
315
35
35
0

Here the Units of my firſt *Product* ſtand under Units of the Integer of my *Dividend*; ſo 7, the firſt figure of my *Quot*, is Integer-Units, and .91 the *Decimal*.

35)276.85(7.91
245 · ·
318
315
35
35
0

Here 5, the Units of my firſt *Product*, ſtand under 6, which are *decimal Primes*; therefore 7, the firſt figure of my *Quot*, is *decimal Primes*, and the *Quot* is entirely a *Decimal* .791

35)27.685(.791
245 ·
318
315
35
35
0

Here 5, the Unit of my firſt *Product*, ſtands under *decimal Seconds*; therefore 7, the firſt of the *Quot*, is *decimal Seconds*; ſo the *Quot* is .0791

```
35)2.7685(.0791
   245 ··
   ———
    318
    315
    ———
     35
     35
     ——
      0
```

Next, we ſhall keep the *Dividend* an *Integer*, and allow *decimal* Places to the *Diviſor*, the figures in the *Dividend* and *Diviſor* being ſtill the ſame.

Here, for the one *decimal* place in the *Diviſor*, I cut off one *decimal* place (.5) from the firſt *Product*, and 4, the integral Unit of my firſt *Product*, ſtands under the place of thouſands in the *Dividend*; ſo 7, the firſt figure of the *Quot*, is thouſands.

```
3.5)27685(7910
    24.5··
    ————
     318
     315
     ———
      35
      35
      ——
      0.0
```

N. B. In this caſe 0 remaining after my *Dividend* is exhauſted, I annex a cypher to that *Remainder* anſwering to .5, the one *decimal* place in the *Diviſor*, and divide, which gives 0 to the *Quot*; which, according to the former *Rule*, is ſtill an *Integer* 7910.

Here, for the two *decimal* places .35 in the *Diviſor*, I cut off two (.45) places from my firſt *Product*: So 2, the Unit of the Integer of that *Product*, ſtands under ten thouſands in the *Dividend*; ſo 7, the firſt figure of the *Quot*, (79100) is ten thouſands.

```
.35)27685(79100
    2.45··
    ----
     318
     315
     ---
       35
       35
       --
```

Here my firſt *Product* has no *Integer*; ſo I ſet a cypher on the left hand of the point, repreſenting the Integer-Units of that *Product*; that cypher ſtands under the place of hundreds of thouſands in the *Dividend*; ſo 7, the firſt figure of the *Quot* is hundreds of thouſands.

```
.035)27685(791000
     0.245··
     -----
       318
       315
       ---
         35
         35
         --
          0
```

N. B. In practice it is eaſy to conceive, in one's mind, the cypher on the left hand of the *decimal* point of the firſt Product, without actually ſetting it down.

We ſhall next allow *decimal* places to both *Dividend* and *Diviſor*.

Here 4, the Unit of the Integer of my firſt *Product* ſtands under tens of the Integer of the *Dividend*; therefore 7, the firſt figure of the *Quot*, is tens: So the *Quot* is 79.1

```
3.5)276.85(79.1
    24.5  ·
    ----
     318
     315
     ---
       35
       35
       --
        0 ·
```

Here 4, the Unit of the Integer of the firſt *Product*, ſtands under Units of the Integer of the *Dividend*; ſo 7, the firſt figure of the *Quot*, is Units, and the *Quot* is 7.91

```
3.5(27.685(7.91
    24.5 ··
    ----
     318
     315
     ---
       35
       35
       --
        0
```

Here 4, the Unit of the firſt *Product*, ſtands under *decimal Primes* of the *Dividend*; therefore 7, the firſt figure of the *Quot*, is *decimal Primes*, and the *Quot* is .791

```
3.5)2.7685(.791
    24.5 ··
    ----
     318
     315
     ---
      35
      35
      --
       0
```

Here the Integer-Unit (4) of the firſt *Product*, ſtands under *decimal Seconds* of the *Dividend*; ſo 7, the firſt figure of the *Quot*, is *decimal Seconds*, and the *Quot* is .0791

```
3.5).27685(.0791
     24.5··
     ————
       318
       315
      ————
        35
        35
        ——
         0
```

Whoever conſiders ſeriouſly the preceeding operations, may eaſily draw from them the following

> **RULE.** *In* Diviſion *of* Decimals, *the place of the firſt figure of the* Quot, *is the ſame with the place of that figure in the* Dividend, *which ſtands above the Units of the* Product *of the* Diviſor, *and firſt figure of the* Quot.

INFI-

INFINITE DECIMALS.

INFINITE DECIMALS are ſuch as repete either the ſame figure, or the ſame circle of figures, infinitely.

Hence they are diſtinguiſhed into *infinite repeting*, and *infinite circulating Decimals*.

Infinite repeting Decimals ariſe from the firſt prime number 3, and its Compounds as *Denominators*.

Infinite circulating Decimals ariſe from the two prime numbers 7 and 11, and their Compoſites.

Here I ſhall lay down the *Decimals* of the *vulgar Fractions* that have for *Denominator* any of the eight digits, 2, 3, 4, 5, 6, &c. or ſome of their compoſites, and make ſuch remarks upon them as may, in ſome meaſure, enable me to account for the Arithmetick of *Infinite Decimals*.

$\frac{1}{2} = .5$ *finite*

$\frac{1}{4} = .25$ *finite*

$\frac{1}{8} = .125$ *finite*

Here it appears, that the *Denominator* 2, affords a *finite Decimal*, and all its compoſites do the ſame, provided their other *factor* or *factors* are not ſuch *Denominators* as produce *Infinites*.

$\frac{1}{3} = .33\dot{3}$

$\frac{2}{3} = .66\dot{6} = \frac{1}{3} + \frac{1}{3}$

3 as a *Denominator* gives an *infinite repeting Decimal*.

CORO-

COROLARY I.

Hence, becauſe $\frac{1}{3} = .333$, to multiply any number by .333, is to take $\frac{1}{3}$ of that number.

COROLARY II.

Becauſe $\frac{2}{3} = .666$, to multiply any number by .666 is to take $\frac{1}{3}$ of that number twice.

$\frac{1}{6} = .1666$
$\frac{2}{6} = .3333 = \frac{1}{3}$
$\frac{3}{6} = .5$ *finite.*
$\frac{4}{6} = .666 = \frac{2}{3}$
$\frac{5}{6} = .8333$

The *Denominator* 6, produces an *infinite repeting Decimal.*

$\frac{1}{7} = .142857,142857,14$
$\frac{2}{7} = .285714,285714,28$
$\frac{3}{7} = .428571,428571,42$
$\frac{4}{7} = .571428,571428,57$
$\frac{5}{7} = .714285,714285,71$
$\frac{6}{7} = .857142,857142,85$

The *Denominator* 7, produces an infinite circle of ſix figures, whoſe ſum is every where 27 = 9×3, or 9 multiplied by half the number of figures in the circle.

N. B. The *Denominator*, 8, being a compoſite of 2 by 2, muſt produce a *finite Decimal.*

$\frac{1}{9} = .111$
$\frac{2}{9} = .222$
$\frac{3}{9} = .333 = \frac{1}{3}$
$\frac{4}{9} = .444$
$\frac{5}{9} = .555$
$\frac{6}{9} = .666 = \frac{2}{3}$
$\frac{7}{9} = .777$
$\frac{8}{9} = .888$

The *Denominator* 9, gives an *infinite Decimal*, whoſe *Numerator* is a continued repetition of the *Numerator* of the *vulgar Fraction.*

A *Fraction* is nothing elſe but an *Integer* divided by the *Denominator*; thus $\frac{1}{9}$ (= .111) is one whole divided by 9, and $\frac{3}{4}$ (= .75) is 3 wholes divided by 4; if therefore .75 (the *Decimal* of $\frac{3}{4}$) is multiplied by 4, the *Product* muſt give the *Integer* 3, and .111 being *infinite*, reckoning the right hand figure a ninth part, *i. e.* multiplying by 9, and carrying at 9 on the right hand, I have my *Integer*, and conſequently the full value of my Fraction. Thus, I ſay, once 9 is 9, 0 and carry 1; 9 times 1 is 9, and 1 that I carried is 10, 0 and carry 1, &c. I, at laſt, find my *Integer*, or *Numerator*, 1. The ſame way, if I multiply .666 (= $\frac{2}{3}$) by the *Denominator* 3, and carry at 9 on the right hand, I have 2 the *Integer*, or *Numerator*, and conſequently the full value of the Fraction; ſo that I take it as an *Axiom*,

$$\begin{array}{r} .111 \\ 9 \\ \hline 1.000 \\ \hline \end{array}$$

$$\begin{array}{r} .666 \\ 3 \\ \hline 2.000 \\ \hline \end{array}$$

The right hand figure of every *infinite repeting Decimal* is ninth parts.

N. B. In *Finite Decimals*, which are limited and equivalent to their *vulgar Fraction*, we carry at 10 on the right hand, whereas in *Infinites* we carry at 9 on the right hand; therefore *Infinites*, in order to their Addition or Subtraction, muſt have, at leaſt, one *decimal* place more than the longeſt *Finite*, and muſt all have an equal number of places amongſt themſelves.

$\frac{1}{11} = .09,09,09$
$\frac{2}{11} = .18,18,18$
$\frac{3}{11} = .27,27,27$
$\frac{4}{11} = .36,36,36$
$\frac{5}{11} = .45,45,45$
$\frac{6}{11} = .54,54,54$
$\frac{7}{11} = .63,63,63$
$\frac{8}{11} = .72,72,72$
$\frac{9}{11} = .81,81,81$
$\frac{10}{11} = .90,90,90$

The *Denominator*, 11, produces an infinite circle of two figures, whoſe ſum is every where 9, or 9 multiplied by half the number of figures in the circle; for $9 \times 1 = 9$

$\frac{1}{12} = .0833$
$\frac{2}{12} = .166 = \frac{1}{6}]\, \frac{3}{12} = \frac{1}{4}]\, \frac{4}{12} = \frac{1}{3}$
$\frac{5}{12} = .4166]\, \frac{6}{12} = \frac{1}{2}$
$\frac{7}{12} = .5833]\, \frac{8}{12} = \frac{2}{3}]\, \frac{9}{12} = \frac{3}{4}$
$\frac{10}{12} = .8333$
$\frac{11}{12} = .9166$

The *Denominator* 12 being a Compound of 3, gives an *infinite repeting Decimal.*

$\frac{1}{13} = .076923,076923,07$
$\frac{2}{13} = .153846,153846,15$
$\frac{3}{13} = .230769,230769,23$
$\frac{4}{13} = .307692,307692,30$
$\frac{5}{13} = .384615,384615,38$
$\frac{6}{13} = .461538,461538,46$
$\frac{7}{13} = .538461,538461,53$
$\frac{8}{13} = .615384,615384,61$
$\frac{9}{13} = .692307,692307,69$
$\frac{10}{13} = .769230,769230,76$
$\frac{11}{13} = .846153,846153,84$
$\frac{12}{13} = .923076,923076,92$

Here again we have a continued revolution of a circle of ſix fifiures, whoſe ſum is 27 = 9×3

$\frac{1}{14} = .0,714285,714285,71\,]\,\frac{2}{14} = \frac{1}{7}$
$\frac{3}{14} = .2,142857,142857,14\,]\,\frac{4}{14} = \frac{2}{7}$
$\frac{5}{14} = .3,571428,57$ &c.$\,]\,\frac{6}{14} = \frac{3}{7}\,]\,\frac{7}{14} = \frac{1}{2}\,]\,\frac{8}{14} = \frac{4}{7}$
$\frac{9}{14} = .6,428571,428571,42\,]\,\frac{10}{14} = \frac{5}{7}$
$\frac{11}{14} = .7,857142,857142,85\,]\,\frac{12}{14} = \frac{6}{7}$
$\frac{13}{14} = .9,285714,285714,28$

Here the *Denominator*, 14, affords a circle of 6 figures, as before, only the figure next the *decimal* point does not enter the circle: This, or more figures, that do not enter the circle, we ſhall call the finite part of the *Circulate*.

$\frac{1}{15} = .0666$ being a multiple of 3, it is *infinite repeting*.

$\frac{1}{16} = .0625$ being a multiple of 2, it is *finite*.

$\frac{1}{17} = .0588235294117647,058,$ &c.
$\frac{2}{17} = .1176470588235294,117$
$\frac{3}{17} = .1764705882352941,176$
$\frac{4}{17} = .2352941176470588,235$
$\frac{5}{17} = .2941176470588235,294$
$\frac{6}{17} = .3529411764705882,352$
&c.

Here we have a continued repetition of a circle of 16 figures, and the ſum of the figures in the circle is 72 = 9×8, or 9 multiplied by $\frac{1}{2}$ the number of figures in the circle.

$\frac{1}{18} = .0555 \,]\, \frac{2}{18} = \frac{1}{9} \,]\, \frac{3}{18} = \frac{1}{6} \,]\, \frac{4}{18} = \frac{2}{9}$
$\frac{5}{18} = .2777 \,]\, \frac{6}{18} = \frac{1}{3}$
$\frac{7}{18} = .3888 \,]\, \frac{8}{18} = \frac{4}{9} \,]\, \frac{9}{18} = \frac{1}{2} \,]\, \frac{10}{18} = \frac{5}{9}$
$\frac{11}{18} = .6111 \,]\, \frac{12}{18} = \frac{2}{3}$
$\frac{13}{18} = .7222 \,]\, \frac{14}{18} = \frac{7}{9} \,[\, \frac{15}{18} = \frac{5}{6} \,]\, \frac{16}{18} = \frac{8}{9}$
$\frac{17}{18} = .9444$

From the *Denominator* 18, ariſe *repeting Infinites*, 18 being a multiple of 2 and 9, or 3 and 6.

$\frac{1}{19} = .052631578947368421,05263,$ &c.
$\frac{2}{19} = .105263157894736842,10526$
$\frac{3}{19} = .157894736842105263,15789$
$\frac{4}{19} = .210526315789473684,21056$

Here 19 (=10+9) affords a circle of 18 figures, whoſe ſum is 81=9×9=9×$\frac{1}{2}$ the number of figures in the circle.

$\frac{1}{20} = .05$
$\frac{2}{20} = .1$
$\frac{3}{20} = .15$
$\frac{4}{20} = .2$

The *Denominator* 20 (being a compoſite of 4 and 5) affords a *finite Decimal*, and univerſally all the powers of 5, and compoſites of 5 and 2, or the powers of 2, afford *finite Decimals*.

$\frac{1}{21} = .047619,047619,04$ &c.
$\frac{2}{21} = .095238,095238,09$
$\frac{3}{22} = .142857,142857,14$

The *Denominator* 21 (being a compoſite of 3 and 7) gives a circle of ſix figures, as before.

$\frac{1}{22} = .0,45,45,45$
$\frac{2}{22} = .0,90,90,90$
$\frac{3}{22} = .1,36,36,36$

The *Denominator* 22, being a compoſite of 2 and 11, gives a circle of two figures, with a *finite* part.

$\frac{1}{23} = .0434782608695652173913,043$
$\frac{2}{23} = .0869565217391304347826,086$

The circle has 22 figures; their ſum is 99 = 9×11

$\frac{1}{26} = .0,384615,384615,38$ &c.
$\frac{2}{26} = .0,769230,769230,76$
$\frac{3}{26} = .1,153846,153846,15$

The *Denominator* 26 (=2×13) affords a circle of ſix figures, with a *finite* part.

$\frac{1}{27} = .037,037,037$
$\frac{2}{27} = .074,074,074\,]\,\frac{3}{27} = \frac{1}{9}$
$\frac{4}{27} = .148,148,148$
$\frac{5}{27} = .185,185,185\,]\,\frac{6}{27} = \frac{2}{9}$
$\frac{7}{27} = .259,259,259$
$\frac{8}{27} = .296,296,296\,]\,\frac{9}{27} = \frac{1}{3}$
$\frac{10}{27} = .370,370,370$
$\frac{11}{27} = .407,407,407\,]\,\frac{12}{27} = \frac{4}{9}$
$\frac{13}{27} = .481,481,481$
$\frac{14}{27} = .518,518,518\,]\,\frac{15}{27} = \frac{5}{9}$
$\frac{16}{27} = .592,592,592$
$\frac{17}{27} = .629,629,629\,]\,\frac{18}{27} = \frac{6}{9}$
$\frac{19}{27} = .703,703,703$
$\frac{20}{27} = .740,740,740\,]\,\frac{21}{27} = \frac{7}{9}$
$\frac{22}{27} = .814,814,814$
$\frac{23}{27} = .851,851,851\,]\,\frac{24}{27} = \frac{8}{9}$
$\frac{25}{27} = .925,925,925$
$\frac{26}{27} = .962,962,962$

The *Denominator* 27, being a compound of 3 and 9, affords a circle of three figures; their ſum is 9+ the *Numerator* of the *vulgar Fraction*, from which the *Decimal* ariſes, if the *Numerator* is a ſimple digit.

But if the *Numerator* of the *vulgar Fraction* exceed 9 or 18, the ſum of the figures in the circle is 9+that exceſs.

$\frac{1}{28} = .03,571428,571428,57,$ &c.$\,]\,\frac{2}{28} = \frac{1}{14}$
$\frac{3}{28} = .10,714285,714285,71\,]\,\frac{4}{28} = \frac{1}{7}$
$\frac{5}{28} = .17,857142,85\,]\,\frac{6}{28} = \frac{3}{14}\,]\,\frac{7}{28} = \frac{1}{4}\,]\,\frac{8}{28} = \frac{2}{7}$
$\frac{9}{28} = .32,142857,142857,14\,]\,\frac{10}{28} = \frac{5}{14}$
$\frac{11}{28} = .39,285714,285714,28\,]\,\frac{12}{28} = \frac{3}{7}$
$\frac{13}{28} = .46,428571,428571,42\,]\,\frac{14}{28} = \frac{1}{2}$
$\frac{15}{28} = .53,571428,571428,57\,]\,\frac{16}{28} = \frac{4}{7}$
$\frac{17}{28} = .60,714285,714285,71\,]\,\frac{18}{28} = \frac{9}{14}$
$\frac{19}{28} = .67,857142,85\,]\,\frac{20}{28} = \frac{5}{7}\,]\,\frac{21}{28} = \frac{3}{4}\,]\,\frac{22}{28} = \frac{11}{14}$
$\frac{23}{28} = .82,142857,142857,14\,]\,\frac{24}{28} = \frac{6}{7}$
$\frac{25}{28} = .89,285714,285714,28\,]\,\frac{26}{28} = \frac{13}{14}$
$\frac{27}{28} = .97,142857,142857,14$

$\frac{1}{19} =$

$\frac{1}{29}$ = .0344827586206896551724137931,03
$\frac{2}{29}$ = .0689655172413793103448275862,06
&c.

From the whole Reduction of *Decimals* already laid down, I ſhall draw three *Obſerves*.

1. Every *vulgar Fraction* may be reduced to a *Decimal*, which is either *finite*, *infinite repeting*, or *infinite circulating*.

For as *vulgar Fractions* are reduced to *Decimals*, by dividing the *Numerator* (with cyphers annexed) by the *Denominator*, the *Remainder* muſt, at laſt, be either 0, and then the *Fraction* is *finite*, or the *Remainder* muſt be always the ſame number, and conſequently the *Quot* the ſame, and then it is *infinite repeting*; or, becauſe the *Diviſor* is a *finite* number, the *Remainder* muſt, at laſt, be the ſame with the *Numerator* of the *vulgar Fraction*, or the ſame with ſome preceeding *Remainder*; and then a new circle begins, and the *Decimal* is *infinite circulating*.

2. Becauſe in *Circulates*, the circle ſometimes runs to 16, 18, 22, 28, &c. figures, we may, without any ſenſible error, limit theſe *Decimals* at 7 (or more) places, and uſe them as *Finites*; for 7 *decimal* places divide the *Integer* into 10000000 equal parts, and all the loſs ſuſtained by thus limiting the *Decimal* is a Fraction of one ten millioneth part of the *Integer*.

3. That tho' in general the ſum of the figures in the circle is equal to 9, multiplied by $\frac{1}{2}$ the

number of figures in the circle, yet that *Rule* is not univerſal.

Infinite repeting DECIMALS.

I SHALL begin my Arithmetick of *Infinite repeting Decimals* with one particular *Rule.*

RULE. *To continue any* Decimal *of a Pound of three places till it is limited,* i.e. *till it is determined whether it is* finite *or* infinite: *If the two figures furtheſt from the* decimal *point are leſs than* 24, *multiply them by* 4, *the* Product (*if leſs than* 24) *gives two places more in your* Decimal; *if the* Product *is* 24, *or more, add* 1; *if* 48, *or more, add* 2; *if* 72, *or more, add* 3 *for the two following places. If the two figures furtheſt from the point exceed* 25, *or* 50, *or* 75, *multiply the exceſs by* 4, *as before; continue this multiplication of the two laſt figures by* 4, *and adding, as above, till you at laſt terminate either in* 25, 50, *or* 75, *and then your* Decimal *is limited and* finite; *or in a repeting* 3, *or a repeting* 6, *and then it is limited and* infinite.

The reaſon of the *Rule* is plain from the reaſon of *Rule* IV. *page* 156, and the reaſon of *Rule* V. *page* 157, in *Reduction of* Decimals.

The *Examples* of this *Rule* will afford *Examples* of *Addition* and *Subtraction* of *infinite repeting*

peting Decimals; ſo I ſhall, firſt of all, from the *Obſerves* already made, lay down a few

DIRECTIONS *for regulating* Infinite Repeters.

1. They muſt not be limited under three *decimal* places.

2. Nor then, unleſs they have, at leaſt, one *repeting* figure.

3. In *Addition* and *Subtraction* they muſt exceed the longeſt *Finite*, by, at leaſt, one place, and muſt all have the ſame number of places amongſt themſelves.

4. In *Addition* and *Multiplication*, for every 9 on the right hand carry 1 to the next place; and in *Subtraction* borrow 9 on the right hand.

N. B. The ſums, differences, and products of *Infinites* are always *infinite*, unleſs they terminate in a cypher.

Examples of the preceeding *Rule*, which ſerve as an *Example* of *Addition*, and afford *Examples* of *Subtraction*.

L.	*s.*	*d.*	*f.*		
82	10	6	3	=	82.528125
66	14	1	1	=	66.7052083
72	15	4	2	=	72.76875
98	10	7	3	=	98.5322916
65	13	4	0	=	65.6666666
79	18	8	0	=	79·9333333
466	02	8	1	=	466.1343750

In *Example* 1. the ſecond and third figures from the point being 28, (which is more than 25) I multiply the exceſs above 25 (*viz.* 3) by 4, and the *Product*, 12, gives 2 places more in the *Decimal*; this 12 (being leſs than 24) I multiply by 4; to the *Product* 48 I add 2, which makes 50, for 2 more figures in my *Decimal*; ſo my *Decimal* is limited and *finite.* I neglect the cypher on the right hand by *N. B.* 1. on the Definition of *Decimals.*

In *Example* 2. the ſecond and third figures from the point, *viz.* 05, I multiply by 4; the *Product* 20, gives two figures more in the *Decimal*, which (20) being leſs than 25, I multiply by 4; to the *Product* 80, (being more than 75) I add 3, which makes 83, two figures more in my *Decimal*; the exceſs of 83 above 75, *viz.* 8, I multiply by 4; to the *Product* 32 I add 1, which makes 33, two figures more in my *Decimal*; the exceſs of 33 above 25, *viz.* 8, multiplied by 4, gives again 33: So my *Decimal*, terminating in a repeting 3, is limited and *infinite.*

In *Addition* I give all my *Infinites* an equal number of places amongſt themſelves, and one place more than .528125, the longeſt *Finite*; then ſay, 3 and 6 make 9, and 6 make 15, and 3 make 18, *viz.* 2 nines preciſely; ſo I ſet down 0, and carry 2 to the next place: I proceed in the reſt as in *Integers.*

Examples of SUBTRACTION.

L.	*s.*	*d.*	*f.*	
82	10	6	3	= 82.528125
66	14	1	1	= 66.7052083
15	16	5	2	= 15.8229166

Here, on the right hand, I ſay, 3 from 0 I cannot, but 3 from 9, and 6 remains, &c.

L.	*s.*	*d.*	*f.*	
82	14	1	1	= 82.7052083
25	10	7	3	= 25.5322916
57	03	5	2	= 57.1729166

Here I ſay, 6 from 3, I cannot, but 6 from 12, (=9+3) and 6 remains.

Examples of MULTIPLICATION.

Here, in multiplying, I carry at 9 on the right hand, and before I add, I bring all my *Infinites* to have their right hand figures directly below each other, and, in adding, carry at 9 on the right hand.

```
    27.8833
        7.5
-----------
    1394166
   19518333
-----------
  209.12500
```

N. B. If your *Multiplicand* is *infinite*, and your *Multiplier* has cyphers on the right hand, multiply the ſignificant figures only; and, inſtead of annexing the cyphers to the right hand of the *Product*, repete the right hand figure of the *Product* as often as you have cyphers.

79.666	874.333	84.777
500	900	800
39833.333	786900.000	67822.222

Here, to the *Product* 39833.3, I annex 33 for the two 00.

What is the price of 1000 ſtones of hay, at 4 *d.* 1 *f.* *per* ſtone?

4 *d.* 1 *f.* = .0177083

.0177083
1000

Anſwer, 17 *l.* 14 *s.* 2 *d.* 17.7083333

N. B. 2. If the *Multiplier* has a *repeting Infinite*, ſet the *repeting* figures beyond the right hand figure of the *Multiplicand*, as we do with cyphers in multiplication of *Integers*; and if the *repeting* figure is 3, firſt take $\frac{1}{3}$ of the *Multiplicand*, and then multiply your other figures: If it is 6, take $\frac{1}{3}$ of the *Multiplicand* twice, and then multiply your other figures; but if it is any other than 3 or 6, firſt multiply that *repeting* fi-

fiure into $\frac{1}{9}$ of the *Multiplicand*, and then multiply your other figures. In all theſe three caſes allow no *decimal* place to the *Product* for the *repeting* figure or figures in the *Multiplier*.

EXAMPLE I.

Here, becauſe of the repeting 333, I take $\frac{1}{3}$ of the *Multiplicand* = 24275; then I multiply my two other figures into the *Multiplicand*, firſt 8, ſetting the Units of the *Product* under Units of the *Multiplicand*; then I multiply 6 into the *Multiplicand*, as in *Integers:* The ſum of $\frac{1}{3}$ the *Multiplicand*, and theſe two *Products*, gives the product of the whole *Multiplication*; from which I cut off three *decimal* places, *viz.* two places for the two (.25) in the *Multiplicand*, and one for the one multiplying *decimal* figure in the *Multiplier:* But I allow no *decimal* place for the repeting figures in the *Multiplier*; for, inſtead of multiplying by .0333, I took $\frac{1}{3}$ of the *Multiplicand*.

```
 3) 728.25
      6.8333
 -----------
1/3=24275
   582600
  436950
 -----------
  4976.375
 -----------
```

E X-

EXAMPLE. II.

Here, in taking $\frac{1}{3}$ of the *Multiplicand*, 1 remains, which I continue to three places, by annexing cyphers to the *Remainder*; ſo $\frac{1}{3}$ the *Multiplicand* = 265.333, and in placing my *decimal* point in the *Product*, I allow three *decimal* places for the three figures in the continuation of my *Diviſion*, and one place for the one multiplying *decimal* figure in the *Multiplier*; ſo my *Product* has four *decimal* places, tho' I have ſtill allowed no *decimal* place to the *Product* for the repeting 3 in the *Multiplier*.

$$\begin{array}{r} 3)796 \\ 2.6333 \\ \hline \frac{1}{3}=265333 \\ 4776 \\ 1592 \\ \hline 2096.1333 \end{array}$$

EXAMPLE III.

Here I ſet my repeting figures .333 beyond the right hand figure of my *Multiplicand*, and o below it, to ſupply the place of an *Integer*, or any other figure in the *Decimal* of the *Muliplier*; then I take $\frac{1}{3}$ the *Multiplicand*, *viz.* 292.5 for my *Product*.

$$\begin{array}{r} 3)\ 877.5 \\ 0.333 \\ \hline \frac{1}{3}=292.5 \\ \hline \end{array}$$

EXAMPLE IV.

Here, for my three figures, (.666) ariſing from the continuation of my *Diviſion*, I allow three *decimal* places to the *Product*.

$$\begin{array}{r} 3)\ 368 \\ 0.333 \\ \hline \frac{1}{3}=122.666 \end{array}$$

EXAMPLE V.

Here, for my repeting 6, I take $\frac{1}{3}$ of the *Multiplicand* twice, then I multiply my other figures, ſetting Units of the firſt *Product* under Units of the *Multiplicand*, &c. I allow no *decimal* place in the *Product* for my repeting 6, becauſe, inſtead of multiplying by it, I took $\frac{2}{3}$ of the *Multiplicand.*

```
3)  680.748
     3.21666
 ─────────────
⅓=226916
⅓=226916
   680748
 1361496
2042244
 ─────────────
2189.73940
 ─────────────
```

EXAMPLE VI.

Here I give the *Product* ſix *decimal* places, *viz.* two for the two in the *Multiplicand*, one for the one multiplying *decimal* place in the *Multiplier*, and three for the three places the continuation of my *Diviſion* went beyond the right hand figure of my *Multiplicand.*

```
3)  75.46
     2.4666
 ─────────────
⅓=2515333
⅓=2515333
   30184
  15092
 ─────────────
186.134666
```

EXAMPLE VII.

Here, becauſe $.555 = \frac{5}{9} = 5\times\frac{1}{9}$, I multiply my repeting figure, 5, into $\frac{1}{9}$ of the *Multiplicand*, and then multiply my other figures. I allow no *decimal* place to the *Product* for my repeting 5; but, becauſe the continuation of my *Diviſion* runs three places beyond the right hand figure of the *Multiplicand*, I muſt allow theſe three places to the *Product*.

$$\begin{array}{r}
9)\,7989 \\
47{\cdot}555 \\
\hline
\frac{1}{9}=887666 \\
5 \\
\hline
4438333 \\
55923 \\
31956 \\
\hline
379921{\cdot}333 \\
\hline
\end{array}$$

N. B. It is plain the *Product* would have come out the ſame, if I had firſt multiplied by 5, and then taken $\frac{1}{9}$ of that *Product*.

EXAMPLE VIII.

Here I take $\frac{1}{9}$ of the *Multiplicand* for my *Product*, becauſe $.111 = \frac{1}{9}$

$$\begin{array}{r}
9)68.85 \\
0.111 \\
\hline
\frac{1}{9}=7.65 \\
\hline
\end{array}$$

The *Rule* is a natural conſequence of the *Obſerves* made on the *Decimals* ariſing from the *Denominators* 3, 6, 9.

QUE-

QUESTIONS.

1. What is the price of 648½ yards of cloth, at 13 *s.* 4 *d. per* yards?

Anſwer 432 *l.* 6 *s.* 8 *d.*

3) 648.5
0.666

$\frac{1}{3}$ = 2161666
$\frac{1}{3}$ = 2161666

432.3333

2. In 456½ Ducates, how many Pounds *Sterling*, at 6 *s.* 2 *d. per* Ducate?

Anſwer 140 *l.* 15 *s.* 1 *d.*

3) 456.5
.30833

$\frac{1}{3}$ = 152166
36520
13695

140.754166

3. In 10363½ Merks *Scots*, how many Pounds *Sterling?* *N. B.* 1 Merk = $\frac{1}{18}$ *l. ſter.* = .055

Anſwer 575 *l.* 15 *s.*

9) 10363.5
.055

$\frac{1}{9}$ = 11515
5

575.75

DIVISION

DIVISION *of* Infinite repeting Decimals.

If your *Diviſor* is *finite*, and your *Dividend infinite*, in continuing your work, inſtead of annexing cyphers to the *Remainder*, annex the *repeting* figure of the *Dividend*.

EXAMPLE.

How many bolls of meal for 561 *l.* 11 *s.* 8 *d. Scots*, at 5 *l.* 15 *s. Scots per* boll?

Here when my *Dividend* was exhauſted, I had but one *decimal* place in my *Quot*, ſo I continue my *Diviſion*, by annexing to the *Remainder* the *repeting* figure of the *Dividend*, 3, inſtead of the cypher.

```
5.75)561.583(97.666
      5175''
      ----
       4408
       4025
       ----
        3833
        3450
        ----
         3833
         3450
         ----
          3833
          3450
          ----
           383
           ----
```

If your *Diviſor* is *infinite*, in continuing your *Diviſion*, inſtead of annexing cyphers to the *Remainder*, annex the right hand figure of the *Remainder*.

EXAMPLE.

```
6.833)758.816(111.051707
      6833 ..
      ----
       7551
       6833
       ----
        7186
        6833
        -----
         35333
         34166
         -----
          11666
           6833
          -----
           48333
           47833
           -----
            50000
            47833
            -----
             2166
             ----
```

The reaſon is plain; for the *Diviſor* being *infinite*, all its products are *infinite*, and their *Remainders*, when ſubtracted from their correſpondent *Dividuals*, are *infinite*.

Here I ſhall add a few more *Examples* of theſe *Rules*, both for the further illuſtration of the *Rules*, and the eaſy application of them to queſtions of different ſorts.

EXAMPLE I.

To divide 684 *l.* 12 *s.* 8 *d.* amongſt 5 men, ſo as 4 ſhall have equal ſhares, and the 5th only half a ſhare.

$\frac{1}{2} = .5$; ſo the *Diviſor* is 4.5

```
4.5)684.633(152.1,407,4
      45 ' ' ' '
      ——
      234
      225
      ———
        96
        90
        ——
         63
         45
         ——
         183
         180
         ———
          333
          315
          ———
           183
           180
           ———
             3
```

EXAMPLE II.

Divide 486 *l.* 16 *s.* 8 *d.* amongſt 4 men, ſo as 3 ſhall have equal ſhares, and the 4th only ¾ of a ſhare.

¾ = .75; ſo the *Diviſor* is 3.75

3.75)486.833(129.822
375
———
1118
750
———
3683
3375
———
3083
3000
———
833
750
———
833
750
———
83

EXAMPLE III.

88 men divide 8469 *l.* 3 *s.* amongſt them, ſo as 87 of them have equal ſhares, and one of them only $\frac{1}{3}$ ſhare.

$\frac{1}{3}$ = .333; ſo the *Diviſor* is 87.333

```
87.333)8469.150(96.975
        786000
        ------
         609150
         524000
         ------
          851500
          786000
          ------
           655000
           611333
           ------
            436666
            436666
            ------
               000
```

EXAMPLE IV.

41 Men, in company, gain 257 *l.* 13 *s.* 4 *d.* of which 39 are to have equal ſhares, and the other 2 are to have, one an half, the other a third ſhare.

$\frac{1}{2} = .5$
$\frac{1}{3} = .333$

$\frac{1}{2}+\frac{1}{3} = .833$

So the *Diviſor* is 39.833

39.833)257.666(6.4686
239000

186666
159333

273333
239000

343333
318666

246666
239000

7666

In caſe your *Diviſor* is an *Infinite*, repeting the ſame ſimple figure, as .111, .333, .555 take the following *Notanda*.

N. B. 1. If your *Diviſor* is .333, multiply the *Dividend* by 3, and you have the *Quot*.

Thus 468.433 divided by .333 = 1405.3

For .333 = $\frac{1}{3}$, and to divide by $\frac{1}{3}$, is to multiply by the *Denominator* 3.

468.433
3
1405.300

N. B. 2. If your *Diviſor* is .666, multiply the *Dividend* by 3, and divide the *Product* by 2, and you have the *Quot*; ſo 432.333, divided by .666 = 648.5

The reaſon is plain from *vulgar Fractions*; for .666 = $\frac{2}{3}$.

432.333
3
2)1297.000
648.5

Or multiply by 1.5, which is the ſame thing as $\frac{3}{2}$, or 3 divided by 2.

432.333
1.5
2161666
4323333
648.5000

N. B. 3. If the *Diviſor* is .111, multiply the *Dividend* by 9, and you have the *Quot.*

Thus 76.074074 divided by .111=684.666

It is plain; for .111=$\frac{1}{9}$, and to divide by $\frac{1}{9}$, is to multiply the *Dividend* by 9; for the *Numerator*, 1, produces the ſame *Product* in *Multiplication*, and the *Quot* the ſame with the *Dividend* in *Diviſion*.

$$\begin{array}{r} 76.074074 \\ 9 \\ \hline 684.666666 \\ \hline \end{array}$$

If your *Diviſor* is any other repeting figure, (than .333, .666, or .111) as .555, multiply the *Dividend* by 9, and divide the *Product* by the repeting figure, and you have the *Quot*.

Thus 836.333 divided by .555 = 1505.4

For .555=$\frac{5}{9}$ which, as in *vulgar Fractions*, multiplies by the *Denominator* 9, and divides by the *Numerator* 5.

$$\begin{array}{r} 836.333 \\ 9 \\ \hline 5)7527.000 \\ 1505.4 \\ \hline \end{array}$$

Or multiply the *Dividend* by 1.8, which is the ſame with $\frac{9}{5}$.

The ſame way, in dividing by .222, multiply by 9, and divide by 2, or multiply by 4.5=$\frac{9}{2}$, and, in dividing by .444, inſtead of multiplying by 9, and dividing by 4, you may multiply by 2.25=$\frac{9}{4}$. But in multiplying by .777, it is moſt convenient to multiply by 9, and divide by 7; for $\frac{9}{7}$ = 1.285714,28, &c. an *infinite Circulate*. And, in dividing by .888, it will be

$$\begin{array}{r} 836.333 \\ 1.8 \\ \hline 6690666 \\ 8363333 \\ \hline 1505.4000 \end{array}$$

be more expeditious to multiply by 9, and divide by 8; for $\frac{9}{8} = 1.125$.

Of Infinite circulating Decimals.

Circulating Decimals are either *pure* or *mixt*.

Pure, when no ſignificant figure interveens betwixt the *decimal* point and the beginning of the firſt circle; as .36,36,36, &c. $= \frac{4}{11}$, or .571428,571428, $= \frac{4}{7}$, or .0,384615,38,&c. $= \frac{1}{26}$

Mixt, when one or more ſignificant figures ly betwixt the *decimal* point and the beginning of the firſt circle; as .1,36,36,&c. $= \frac{3}{22}$, or .17,857142,857, &c. $= \frac{5}{28}$

N. B. The figures lying betwixt the *decimal* point and the firſt circle, make the *finite* part of the *Circulate*. Thus, in the firſt *Example*, 1 is the *finite* part, and in the other ,17 is the *finite* part.

N. B. I ſeparate the *finite* part from the circle, and the circles from one another, by a *comma*; thus, .17,857142,857142,8, where the *finite* part 17, is ſeparated from the circle, 857142, by a *comma*, and that circle from the next by a *comma*.

N. B. 2. Any *pure Circulate* may be conſidered as *mixt*, by taking one or more circles, or part of one, or one and part of another circle for a *finite* part. Thus, .36,36,36 = .363,63,63, where .363 is the *finite* part, and 63 the circle; the ſame way .571428,571428, = .57,142857,14, &c. where .57, is the *finite* part, and 142857 the circle.

COROLARY.

Hence every *Circulate* may be changed into another *Circulate*, whoſe firſt circle begins at any aſſignable number of places from the *decimal* point.

DEFINITION.

Similar, or like *Circulates*, are thoſe whoſe circles conſiſt of an equal number of figures, and begin at an equal number of places from the *decimal* point.

Reduction of Circulates.

RULE I. To make two or more *Circulates* ſimilar.

> *Make all your Circles begin at the ſame diſtance from the* decimal *point, which diſtance is determined by the number of* decimal *places in the longeſt* finite *part of all your given* Circulates, *and let each circle conſiſt of as many figures as are in your longeſt circle.*

Thus $\frac{11}{14}+\frac{9}{28}=.7,857142,85+.32,142857,142,$&c. $=.78,571428+.32,142857$, where, .7, the *finite* part of the firſt *Circulate*, has but one *decimal* place, and .32, the *finite* part of the other, has two; ſo I give two places, .78, to the *finite* part of the firſt *Circulate*, and ſix figures (,571428) to the circle.

Again, $\frac{6}{11}+\frac{15}{28}=.54,54,+.53,571428,57,$ &c. $.54,545454,+.53,571428$, where I give two *decimal* places (.54) to the *finite* part of the firſt *Circulate*, becauſe the *finite* part of the other *Circulate* has two, and I give ſix figures (.545454) to the circle of the firſt, becauſe the circle of the other *Circulate* has ſix places. Again, $\frac{6}{11}+\frac{16}{27}+\frac{5}{28}=.54,54,+.592,592,+.17,857142=.54,545454+.59,259259+.17,857142$

Here, tho' the firſt two *Circulates* have no *finite* part, yet I allow two places (*viz.* .54, and .59) to the *finite* part of each, becauſe the third *Circulate* has (.17) a *finite* part of two places, and I allow ſix figures to the circle of each, becauſe the circle of the third has ſix figures.

N. B. If, in *Addition* and *Subtraction* you have *finite Decimals* mixt with your *Circulates*, you muſt extend all the *finite* parts of your *Circulates* to as many *decimal* places as there are *decimal* places in your longeſt *finite*. Thus, $\frac{6}{11}+\frac{3}{8}+\frac{15}{28}=.54,54,+.375+.53,571428=.545,454545+.375+.535,714285$

RULE II. To reduce *Infinite Circulates* to *vulgar Fractions*.

1. *If the* Circulate *is* pure, *the* Denominator *of the* vulgar Fraction *conſiſts of as many Nines as there are figures in the circle ; for all* infinite Decimals *are* 9th *parts.*

So $\frac{4}{11}=.36,36=\frac{36}{99}$

And $\frac{3}{7}=.428571,42=\frac{428571}{999999}$

2. *If your* pure Circulate *has cyphers betwixt the* decimal *point and circle, make the circle ſtill the* Numerator *of your* vulgar Fraction, *and join as many cyphers to the right hand of the* Denominator *of the circle as there are cyphers betwixt the circle and* decimal point.

So $\frac{1}{22}$=.0,45,45=$\frac{045}{990}$=$\frac{45}{990}$

And $\frac{1}{14}$=.0,714285=$\frac{714285}{9999990}$

3. *If your* Circulate *is* mixt, *multiply the* finite *part by the* Denominator *of the circle, to the* Product *add the circle for the* Numerator *of your* vulgar Fraction, *and join as many cyphers to the right hand of the* Denominator *of the circle as there are figures in the* finite *part, for its* Denominator.

So $\frac{9}{28}$=.32,142857=$\frac{32142825}{99999900}$

Here I multiply the *finite* part (32) by 999999, the *Denominator* of the circle, as directed in *page* 20, and to that *Product* I add the circle 142857, and have 32142825 the *Numerator* of my *vulgar Fraction*, whoſe *Denominator* is 99999900.

32000000
32
———
31999968=32×999999
142857= *Circle.*
———
32142825= *Numer.*

The ſame way $65\frac{8}{27}=65.296,296,=\frac{65231}{999}$

$$\begin{array}{r}65000\\65\\\hline 64935\\296\\\hline 65231\\\hline\end{array}$$

And $36\frac{9}{14}=36.6,428571=\frac{366428205}{9999990}$

$$\begin{array}{r}366000000\\366\\\hline 365999634\\428571\\\hline 366428205\\\hline\end{array}$$

Addition of Circulates.

Make all your *Circles* ſimilar; find the number of Tens in the ſum of the left hand column of your *Circles*, and if the ſum comes to one, two, &c. Tens, and near to one Ten more, to that ſum add the Tens in the ſum of the column immediately on the right hand of the left hand column; if by this addition you make the other Ten, reckon it one Ten more in your left hand column; add theſe Tens to the ſum of the right hand column of your *Circles*, and proceed in the reſt as in *Integers*.

EXAMPLE I.

$\frac{3}{7} = .428571,42 \quad = .4,285714$

$\frac{4}{11} = .36,36 \quad = .3,636363$

$\frac{10}{14} = .7,142857,14 = .7,142857$

$\frac{8}{27} = .296,296 \quad = .2,962962$

$1.8,027898$

Here the ſum of (9+1+6+2) the left hand column of my *Circles* is 18, which wants but 2 of making two Tens; therefore I find the ſum of (6+4+3+8) its immediate right hand column, which affords me two Tens, which two Tens, added to (18) the ſum of my left hand column, make 20, or two Tens, theſe I carry to the right hand column (2+7+3+4) of my *Circles*, and ſay, 2, that I collected out of my left hand column, and 2 make 4, and 7 make 11, and 3 make 14, and 4 make 18, I ſet down 8 and carry 1. I work in the reſt as in *Integers.*

EXAMPLE II.

$\frac{3}{7} = .428571, \quad = .42,857142$

$\frac{3}{4} = .75 \quad = .75$

$\frac{4}{11} = .36, \quad = .36,363636$

$\frac{2}{3} = .666 \quad = .66,666666$

$\frac{10}{14} = .7,142857, = .71,428571$

$2.92,316017$

Here the ſum of (4+6+3+8) the left hand column of my *Circles*, is 21, where I have two Tens, and want 9 of three Tens; as the immediate right hand column cannot afford theſe nine Tens, I carry my two Tens, collected out of my left hand column to the right hand column of my *Circles*, and proceed as before.

N. B. Here, becauſe the *finite Decimal* (.75) has two *decimal* places, I make the *finite* part of each *Circle* conſiſt of the ſame number of places.

EXAMPLE III.

$$\begin{array}{lll}
86\frac{5}{7} & = 86.714285, & = 86.714,285714, \\
75\frac{8}{9} & = 75.888 & = 75.888,888888, \\
36\frac{7}{8} & = 36.875 & = 36.875 \\
42\frac{4}{11} & = 42.36,36 & = 42.363,636363 \\
18\frac{5}{14} & = 18.3,571428, & = 18.357,142857 \\
\hline
 & & 260.198,953823 \\
\hline
\end{array}$$

The reaſon of adding to the right hand column of my *Circles* the Tens that are collected out of the two left hand columns, is plain; for the *Circulate* repeting the *Circle* infinitely, if I had ſet down the *Circles* twice, and added them, the Tens in the left hand column of the ſecond *Circles* would naturally fall in to the right hand column of the firſt *Circles*.

For a proof of my laſt Operation I ſhall add all the *vulgar Fractions* in the *Example*, and reduce their ſum to a mixt number, continuing my *Diviſion* to a *Decimal*. If the integer of my mixt number is the ſame with the number of Tens carried from the colume of *Primes* in the *Addition* to the Units of the *Integers*, and the *Decimal* the ſame with the *Decimal* of my ſum, both as to the *finite* and *circular* part, my work is manifeſtly right, and my *Rule* juſt.

$$\frac{5}{7}+\frac{8}{9}+\frac{7}{8}+\frac{4}{11}+\frac{5}{14}=\frac{55440}{77616}+\frac{68992}{77616}+\frac{67914}{77616}+\frac{28224}{77616}+\frac{27720}{77616}=\frac{248290}{77616}=3.198,953823,95 \text{ \&c.}$$

```
77616)248290(3.198,953823,9 &c.
      232848
      ------.
       154420
        77616
       ------.
        768040
        698544
        ------.
         694960
         620928
         ------.
          740320
          698544
          ------.
           417760
           388080
           ------.
            296800
            232848
            ------.
             639520
             620928
             ------.
              185920
              155232
              ------.
               306880
               232848
               ------
                74032
                -----
```

Here the *Remainder* is the ſame with that of the 4th *Dividual*, which begins a new *Circle*.

Here the *decimal* of the *Quot*, is the ſame with the *decimal* in my Addition, and (3) the *integer* of the *Quot*, the ſame with the number of Tens I carried from my column of *Primes* to the column of Units.

After the ſame manner I can prove my former two operations to be juſt; but I think this may ſuffice.

Subtraction of Circulates.

Make your *Circulates* ſimilar, and work as in *Integers*. Only, if the left hand figure of the circle of the *Subtrahend* is greater than the left hand figure of the circle of the *Minuend*, add 1 to the right hand figure of the *Subtrahend*, before you begin to ſubtract.

The reaſon of adding 1 (in the Hypotheſis in the *Rule*) to the right hand figure of the circle of the *Subtrahend* before *Subtraction*, is, if each circle was twice placed in ſubtracting the left hand figure of the ſecond circle of the *Subtrahend* from the left hand figure of the ſecond circle of the *Minuend*, we muſt borrow 10, which muſt be carried as 1 to the right hand figure of the firſt circle of the *Subtrahend*.

EXAMPLE I.

$$\frac{25}{27} = .925{,}925 = .925925$$

$$\frac{4}{7} = .571{,}428 = .571428$$

$$.354497$$

EXAMPLE II.

$\frac{10}{14}$ = .7,142857,14 = .71,428571
$\frac{5}{28}$ = .17,857142,8 = .17,857142

53,571428

In this *Example*, becauſe 8, the left hand figure of the circle of the *Subtrahend*, is greater than 4, the left hand figure of the *Minuend*, I add 1 to 2, the right hand figure of the circle of the *Subtrahend*, and ſay, 1 and 2 make 3, from 1 I cannot, but 3 from 11, and 8 remains, *&c.*

EXAMPLE III.

$\frac{5}{13}$ = .384615,3 = .384,615384
$\frac{1}{8}$ = .125 = .125,

.259,615384

EXAMPLE IV.

$\frac{13}{14}$ = .9,285714 = .9,285714
$\frac{2}{3}$ = .6,666666 = .6,666666

2,619047

EXAMPLE V.

$\frac{3}{4}$ = .75 = .75,000000
$\frac{1}{14}$ = .0,714285,7 = .07,142857

.67,857142

In *Example* V. becauſe the *finite Decimal* (.75) has two places, I give two to the *finite* part of my *Circulate*, and complete my circle; and becauſe 1, the left hand figure of the *Subtrahend*, exceeds 0, the left hand figure of the circle of the *Minuend*, I add 1 to 7, the right hand figure of the *Subtrahend*.

EXAMPLE VI.

$\frac{5}{13} = .384615,3 = .384,615384$

$\frac{1}{8} = .125 \qquad\qquad = .125$

$.259,615384$

EXAMPLE VII.

$\frac{2}{3} = .666 \qquad\qquad = .6,666666$

$\frac{5}{14} = .3,571428,5 = .3,571428$

$.3,095238$

EXAMPLE VIII.

$\frac{13}{14} = .9,285714,2 = .9,285714$

$\frac{2}{3} = .666 \qquad\qquad = .6,666666$

$.2,619047$

The reaſon of the *Rule* for *Subtraction* of *Circulates* is ſo evident, that I think it needleſs to advance any proof of my work, by reducing the difference of any two of my *vulgar Fractions* to a *Decimal*.

Multiplication of Circulates.

If your *Multiplicand* is a *Circulate*, and your *Multiplier finite*, and one figure only; firſt find how many Tens are in the product of the left hand figure of your Circle, then work as in *Integers*, adding to the product of the right hand figure of the Circle the Tens that are in the product of its left hand figure.

EXAMPLE.

$\frac{11}{14}\times4=3.1,428571$

$$\begin{array}{r} \frac{11}{14}=\ .7,857142 \\ 4 \\ \hline 3.1,428571 \\ \hline \end{array}$$

Here the left hand figure of my Circle is 8; ſo I ſay, 4×8=32, where I have three Tens; then I begin on the right hand, and ſay, 4 times 2 make 8, and 3 (which I had from my left hand product) make 11, I ſet down 1 and carry 1, ſaying, 4 times 4 make 16, and 1 that I carried makes 17, &c.

It is plain that $\frac{11}{14}\times4=\frac{44}{14}$, which, being reduced to a *Decimal*, gives 3.1,428571,428571, or 3.142857,142857, &c. for the circle holds either way.

```
14)44(3.1,428571, &c.
   42
   ——
    20
    14
    ——
     60
     56
     ——
      40
      28
      ——
       120
       112
       ———
         80
         70
         ——
         100
          98
          ——
           20
           14
           ——
            6
           ——
```

N. B. If the product of the left hand figure of the circle affords any number of Tens, and comes near to one Ten more, ſee if the Tens of the product of its right hand figure make up that Ten, and, if they do, add that Ten likewiſe to the product of the right hand figure of the circle, and proceed in the reſt as in *Integers.*

EXAMPLE.

$\frac{11}{14}\times 6=4.7,142857$

$$\begin{array}{rr} \frac{11}{14}= & .7,857142 \\ & 6 \\ \hline & 4.7,142857 \\ \hline \end{array}$$

Here I multiply 8, the left hand figure of the circle, by 6, and find the product (48) wants but two of five Tens; therefore I multiply its right hand figure 5 by 6, and have three Tens; then I ſay, 6 times 8 make 48, and and 3 make 51, where I have five Tens; then, beginning at the right hand figure of the circle, I ſay, 6 times 2 make 12, and five Tens on the left hand of the circle make 17, I ſet down 7 and carry 1, &c.

The reaſon of carrying the Tens of the product of the figure next the left hand figure of the circle, to the product of that left hand figure, and adding the Tens in that product of the left hand figure, to the product of the right hand figure of the circle, is, that the ſame circle being repeted, theſe Tens, by the common courſe of *Multiplication*, muſt fall in to the product of the right hand figure of the circle, as in the marginal operation.

$$\begin{array}{r} .7,857142,857142,\&c \\ 6 \\ \hline 4.7,142857,142857 \\ \hline \end{array}$$

If the *Multiplicand* is a *Circulate*, and the *Multiplier* an *infinite repeting Decimal*, repeting one figure only, as .333, or .666, or .555, in dividing (as directed in the Arithmetick of *infinite repeting Decimals*) by 3 or 9; inſtead of ſuppoſing a cypher on the right hand of the remainder of the right hand figure of the circle, conceive the left hand figure of the circle on the right hand of that remainder, and the figure next to the left hand figure on the right hand of that new remainder, &c. Thus,

$\frac{15}{28}\times\frac{1}{3}=.53,571428\times.333=.17,857142,857,\&c$

Here I ſay, 3 in 5 once, and 2 remain, which make (with the next figure 3 on the right hand) 23; then 3 in 23, anſwers ſeven times, and 2 remain, which make 25; then 3 in 25 eight times, and 1 remains; 3 in 17 five times, and 2 remain; then

$$\begin{array}{l} 3).53,571428 \\ \quad .17,857142,85\ \&c \end{array}$$

3 in 21 ſeven times, o remains; then 3 in 4 once, and 1 remains; then 3 in 12 four times, o remains; then 3 in 8 twice, and 2 remain; on the right of this remaining 2, I conceive 5, (the left hand figure of the circle) which makes 25; then 3 in 25 anſwer eight times, &c.

Again, $\frac{15}{28}\times\frac{2}{3}$=.53,571428×.666=.35,714285

Here I take $\frac{1}{3}$ of the *Multiplicand* twice, conformable to the doctrine already laid down here, and in *infinite repeting Decimals.*

$$\begin{array}{r} 3).53,571428 \\ \hline .17,857142,85 \\ .17,857142,85 \\ \hline .35,714285, \\ \hline \end{array}$$

Again, $\frac{15}{28}\times\frac{5}{9}$=.53,571428×.555=.29,761904,

Here I take $\frac{1}{9}$ of the *Multiplicand*, as above directed, and multiply it by 5 the repeting figure, and I have the product .29,761904, where the circle is ,761904,

$$\begin{array}{r} 9).53,571428 \\ \hline .05,952380,95 \\ 5 \\ \hline .29,761904, \\ \hline \end{array}$$

If your *Multiplicand* has a *Circulate*, and the *Multiplier* is a *finite* number, conſiſting of any number of digits, multiply each figure of the *Multiplier* into the whole *Multiplicand*, as before directed, ſetting off the circle in each product with a *comma*; repete the circle of each product till you have a figure directly below the right hand figure of the firſt product, and then conceive all the circles to begin where the circle of the firſt product begins; add as directed in addition of *Circulates*, and you have the product.

Thus $86\frac{15}{28} \times 58\frac{3}{4} = \frac{2423}{28} \times \frac{235}{4} = \frac{569405}{112} \Rightarrow$ $86.53,571428 \times 58.75 = 5083.9732,142857$

Here, to the right hand of my ſecond product, I annex 9, the left hand figure of its circle; to the right hand of my third product I annex 57, the two left hand figures of its circle, and to the right hand of my fourth product, I annex 857, the three left hand figures of its circle; ſo I have the right hand figures of all my products in one column. I make the figures in the ſame column with 8 (the left hand figure of the circle of the firſt product) the initials, or left hand figures of the circles of the other products; ſo that, as ,857142 is the circle of the firſt product, ſo ,999999 is the circle in the ſecond product; ,142857 is the circle in the third, and ,142857 in the fourth; then I begin on the right hand, and add as in addition of *Circulates*, and find my circle ,142857.

$$\begin{array}{r}
86.53,571428 \\
58.75 \\
\hline
43\ 2\ 6\ 7,857142 \\
605\ 7\ 4,9\ 999999 \\
6922\ 8,5\ 7\ 142857 \\
43267,8\ 5\ 7\ 142857 \\
\hline
5083.97\ 3\ 2,142857 \\
\hline
\end{array}$$

```
112)569405(5083.9732,142857, &c.
    560'''
    ---
      940
      896
      ---
       445
       336
       ---
       1090
       1008
       ----
        820
        784
        ---
         360
         336
         ---
          240
          224
          ---
           160
           112
           ---
            480
            448
            ---
             320
             224
             ---
              960
              896
              ---
               640
               560
               ---
                800
                784
                ---
                 160
```

As a proof of my work, I here reduce the product of my two *improper Fractions, viz.* $\frac{569405}{112}$, to a *mixt* number, continuing my *Diviſion* to a *Decimal*, and find it exactly correſponds with my former operation.

If your *Multiplier* has a repeting figure, work as directed in the Arithmetick of *Infinite repeting Decimals*; only, in multiplying by the other figures, remember to add the Tens in the product of the left hand figure of the circle to the product of its right hand figure.

Thus $78\frac{15}{28}\times54\frac{5}{6}=\frac{2199}{28}\times\frac{329}{6}=\frac{723471}{168}=$ $78.53,571428\times54.8333=4306.375$

Here, for my repeting 3, I take $\frac{1}{3}$ of the *Multiplicand*, and then multiply my other figures according to the *Rules* of *Multiplication* of *Circulates* already laid down, and find my product to be 4306.374,999999. Now, becauſe *repeting Decimals* are always ninth parts, this circle, 999999, is the ſame with $\frac{999999}{999999}=1$; this 1 added to .374, the *finite* part of my *Decimal*, makes .375; ſo my product is 4306.375, a *finite Decimal.*

$$\begin{array}{r} 3)\ 78.53,571428 \\ 54.8333 \\ \hline \frac{1}{3}=26.1\ 7,857142,85 \\ 628\,2\ 8,571428 \\ 3141\ 4,2\ 857142 \\ 39267,8\ 5714285 \\ \hline 4306.374,999999 \\ \hline \end{array}$$

Here, to prove my operation to be juſt, I ſhall reduce $\frac{723471}{168}$, the product of my two *improper Fractions*, $\frac{2199}{28}$ and $\frac{329}{6}$, to a *mixt* number, continuing my diviſion to a *Decimal.*

Here the *Quot* preciſely correſponds with the operation.

```
168)723471(4306.375
    672'''
    ---
     514
     504
     ----
      1071
      1008
      ----
        630
        504
        ----
        1260
        1176
        ----
          840
          840
          ---
           00
           --
```

N. B. Hence, if, in multiplication of *Circulates*, the circle of the product is a repeted 9; that circle is equal to Unity, to be added to the *finite* part of the *Decimal*, and makes the product a *finite Decimal.*

Again, $68\frac{15}{28}\times86\frac{5}{9}=\frac{1919}{28}\times\frac{779}{9}=\frac{1494901}{252}=$ 68.53,571428×86.555=5932.14,682539,

9) 68.53,571428
86.555

$\frac{1}{9}$= 7.61,507936, 5
5

---

38 0 7,539682
411 2 1,428571
5482 8,5 714285

5932.1 4,682539

Here I multiply my repeting 5 into $\frac{1}{9}$ the *Multiplicand*, as taught in *repeting Infinites*, and in multiplying by the other figures, and adding the products, I obſerve the *Rules* already laid down in *infinite Circulates*.

As

```
252)1494901(5932.14,682539,6
    1260'''
    ----
     2349
     2268
     ----
       810
       756
       ---
        541
        504
        ---
         370
         252
         ---
         1180
         1008
         ----
          1720
          1512
          ----
           2080
           2016
           ----
             640
             504
             ---
             1360
             1260
             ----
              1000
               756
              ----
              2440
              2268
              ----
               1720
```

As a proof of my work, I reduce my *vulgar Fraction*, $\frac{1494901}{252}$, to a *Decimal*, and find, that in every circumſtance it correſponds with my product; for my *Remainder*, (172) which limits my circle, is the ſame with the remainder which limited the *finite* part; ſo there a new circle muſt begin.

If both *Multiplicand* and *Multiplier* are *Circulates*, reduce both to *vulgar Fractions*, and work accordingly; then reduce the product to a *Decimal*.

EXAMPLE I.

A *pure Circulate* multiplying a *pure Circulate*.

$\frac{5}{7}\times\frac{4}{11}=\frac{20}{77}=.714285\times.36=\frac{714285}{999999}\times\frac{36}{99}=\frac{25714260}{999999\times99}=.259740,$

```
999999)25714260(25.714285
         1999998
         -------
         5714280
         4999995
         -------
          7142850
          6999993
          -------
           1428570
            999999
           -------
           4285710
           3999996
           -------
           2857140
           1999998
           -------
            8571420
            7999992
            -------
             5714280
             4999995
             -------
              714285
```

```
   714285 num.
       36 num.
  -------
  4285710
 2142855
 --------
 25714260 prod.
```

Here I firſt divide (25714260) the product of my *Numerators*, by 999999; the *Remainder* (714285) is the ſame with the *Remainder* which produced the prime of my *Decimal*, and ſo renews the circle.

I again divide the *Quot* (25.714285) by 99, and find the *Remainder* 25, to which if I bring down (7) the left hand figure of the circle of the *Dividend*, I have a new *Dividual*, (257) the ſame with my firſt *Dividual*, which, of conſequence, renews the circle.

```
99)25.714285(.259740,
   198 ,,,,,
   ---
    591
    495
    ---
     964
     891
     ---
      732
      693
      ---
       398
       396
       ---
        25
```

EXAMPLE II.

A *pure Circulate* multiplying a *mixt Circulate.*

$\frac{15}{28}\times\frac{4}{11}=\frac{60}{308}=.53,571428\times.36=\frac{53571375}{99999900}\times\frac{36}{99}=.194805,$

```
  53000000
        53
  --------
  52999947=53×999999
    571428=Circle.
  --------
  53571375=Numerator.
```

Here (as directed in *page* 20) I multiply (53) the *finite* part (of .53,571428) by (999999) the *Denominator* of the circle; to that *Product* (by *Rule*

Rule II. of *Reduction* of *Circulates*) I add the circle for the *Numerator* of my *Fraction*; ſo $\frac{15}{28} = \frac{53571375}{99999900}$: And by the ſame *Rule* II. $.36 = \frac{36}{99}$ their product, $\frac{53571375}{99999900} \times \frac{36}{99} = \frac{1928569500}{99999900 \times 99}$, and dividing both *Numerator* and *Denominator* by 99999900, we have $\frac{19.285714}{99} = .194805,19$ &c

```
              53571375
                    36
              --------
             321428250
            160714125
            ---------
 99999900)1928569500(19.285714,
           99999900
           --------
           928570500
           899999100
           ---------
            285714000
            199999800
            ---------
             857142000
             799999200
             ---------
              571428000
              499999500
              ---------
               714285000
               699999300
               ---------
                142857000
                 99999900
                ---------
                 428571000
                 399999600
                 ---------
                  28571400
                 ---------
```

Here, in my ſecond diviſion, bringing down 2, the left hand figure of the circle of the *Dividend*, to the *Remainder* 19, we have a new *Dividual*, the ſame with the firſt, which, of conſequence, begins a new circle.

99)19.285714(.194805
99
—
938
891
—
475
396
—
797
792
—
514
495
—
19

As a proof of my work, I reduce $\frac{60}{308}$, the product of my two ſimple Fractions, $\frac{15}{28}$ and $\frac{4}{11}$, to a *Decimal*, and find the circle comes out the ſame; for the *Remainder* 60, being the ſame with the *Numerator* of the *Fraction*, renews the circle.

308)60.0(.194805
308
—
2920
2772
—
1480
1232
—
2480
2464
—
1600
1540
—
60

I ſhall here prove my work by reducing $\frac{20}{77}$, the product of ($\frac{5}{7}\times\frac{4}{11}$) the *vulgar Fractions* firſt given to a *Decimal*.

```
77)20.0(.259740,
   154
   ---
    460
    385
    ---
     750
     693
     ---
      570
      539
      ---
       310
       308
       ---
         20
         --
```

Here the circle is the ſame as in my former operation, and the *Remainder* (20) renews the circle.

EXAMPLE III.

A *mixt Circalate* multiplying a *mixt Circulate.*

$\frac{9}{28}\times\frac{15}{22}=\frac{135}{616}=.32,142857,\times.6,81=\frac{32142825}{999999900}$ $\times\frac{675}{990}=.219,155844$

$$\begin{array}{r} 32000000 \\ 32 \\ \hline 31999968 \\ 142857 \\ \hline 32142825 \text{ \textit{numerator.}} \\ \hline \end{array}$$

$$\begin{array}{r} 600 \\ 6 \\ \hline 594 \\ 81 \\ \hline 675 \text{ \textit{numer.}} \\ \hline \end{array}$$

Here

```
              32142825
                   675
             ---------
             160714125
            224999775
           192856950
          ------------
 99999900)2169640687,5,(216.96,428571,
          199999800
          ---------
           169642687
            99999900
           ---------
           696427875
           599999400
           ---------
            964284750
            899999100
            ---------
             642856500
             599999400
             ---------
              428571000
              399999600
              ---------
               285714000
               199999800
               ---------
                857142000
                799999200
                ---------
                 571428000
                 499999500
                 ---------
                  714285000
                  699999300
                  ---------
                   142857000
                    99999900
                   ---------
                    42857100
```

Here the *Remainder* 42857100, being the ſame with that *Remainder* which begun my circle, muſt begin a new circle.

By

```
990)216.96,428571(.219,155844
           ,,,,,,
    1980
    ----
     1896
      990
     ----
      9064
      8910
      ----
       1542
        990
       ----
        5528
        4950
        ----
         5785
         4950
         ----
          8357
          7920
          ----
           4371
           3960
           ----
            4114
            3960
            ----
             154
```

By my firſt *Diviſion* I brought my *Product* to $\frac{216.96,428571}{990}$ which by this ſecond Diviſion comes out .219,155844,155 &c.

I ſhall here prove my work by reducing $\frac{135}{616}$, the product of my *vulgar Fractions* firſt given, to a *Decimal*.

616)135.0(.219,155844,
1232

1180
616

5640
5544

960
616

3440
3080

3600
3080

5200
4928

2720
2464

2560
2464

96

Here the *Remainder* is the ſame with that which begun my circle 155844

EXAMPLE IV.

A *mixt* number, conſiſting of an *Integer* and *Circulate*, multiplying a *mixt* number conſiſting of an *Integer* and *Circulate*.

$78\frac{5}{11}\times8\frac{8}{27}=\frac{863}{11}\times\frac{224}{27}=\frac{193312}{297}=650\frac{262}{297}=$
$78.45\times8.296=\frac{7767}{99}\times\frac{8288}{999}=\frac{64372896}{999\times99}=$
650.882154,882 &c.

```
             7767                  7800
             8288                    78
            ------                 ------
            62136                  7722
           62136                     45
          15534                    ------
         62136                     7767 numerator.
        -------------              ------
 999)64372896(64437.3              8000
     5994  , , , ,                    8
     -----                         ------
      4432                         7992
      3996                          296
      -----                        ------
       4368                        8288 numerator.
       3996                        ------
       -----
        3729
        2997
        -----
         7326
         6993
         -----
          3330
          2997
          -----
           3330
```

By

```
99)64437.333(650.882154,8
        , , , , ,
   594
   ---
    503
    495
    ---
     873
     792
     ---
      813
      792
      ---
       213
       198
       ---
        153
         99
        ---
        543
        495
        ---
         483
         396
         ---
          873
          792
          ---
           81
           --
```

By my firſt *Diviſion* I reduce my product $\frac{64372896}{999 \times 99}$ to $\frac{64437.333}{99}$, and by my ſecond *Diviſion* I bring it to 650.882154,88; for the *Remainder* 81 begins a new circle, as being the ſame with the *Remainder* of my third *Dividual*.

Here I prove my work by reducing $\frac{262}{297}$, the fractional part of the product of the *vulgar Fractions* $8\frac{63}{11}$ and $2\frac{24}{27}$, first given to a *Decimal*, which comes out the ſame.

```
297)262.0(.882154,8
    2376
    ----
     2440
     2376
     ----
       640
       594
       ---
        460
        297
        ---
        1630
        1485
        ----
         1450
         1188
         ----
          2620
          2376
          ----
           244
           ---
```

Diviſion

Diviſion of Circulates.

If your *Dividend* is a *Circulate*, and your *Diviſor finite*, whether *Integer*, *Decimal*, or *mixt* number, work as in *Integers*; only, in continuing your *Diviſion*, inſtead of annexing cyphers to the *Remainder*, repete the figures of the *Circle*.

EXAMPLE I.

An *Integer* dividing a *Circulate*.

$\frac{25.17,857142}{8} = 8)\ 25\frac{5}{28}\ (=\frac{8}{1})\ \frac{705}{28}\ (=\frac{705}{224}$
$=3.14732,142857,142$ *&c.*

Diviſor. *Dividend.*
8) 25.17,857 142
3.14732,142857,14 *Quot.*

```
224)705(3.14732,142857,1
    672
    ---
    330
    224
    ---
    1060
     896
    ----
     1640
     1568
     ----
       720
       672
       ---
        480
        448
        ---
         320
         224
         ---
          960
          896
          ---
           640
           448
           ---
           1920
           1792
           ----
            1280
            1120
            ----
             1600
             1568
             ----
               320
```

Here, for a proof of my work, I reduce $\frac{705}{224}$, the *Quot* of my *vulgar Fraction* divided by 8, to a *Decimal*, and find it comes out the ſame with my *Quot* when I divide the *Circulate* 25.17,857142 by 8.

E X-

E X A M P L E. II.

A *finite Decimal* dividing a *Circulate.*

$\frac{.9,28\,5714}{.4} = \frac{2}{5})\frac{13}{14}(= \frac{65}{28}) = .232,142857,14$

Diviſor. *Dividend.*

.4) .9,28 5714

.2 32,142857,14 *Quot.*

E X A M P L E III.

A *mixt* number conſiſting of an *Integer* and *finite Decimal*, dividing a *mixt* number having a *Circulate.*

$\frac{25.0,714285,7}{6.5} = 6\frac{1}{2})25\frac{2}{28}(= \frac{13}{2})\frac{702}{28}(= \frac{1404}{364}$ $=3.857142,$

6.5)25.0,714285,7(3.857142,8
195
557
520
371
325
464
455
92
65
278
260
185
130
557

Here the *Remainder*, 55, being the ſame with that of the firſt *Dividual*, renews the *Circle*.

I ſhall here prove my work by reducing $\frac{1404}{364}$, the *Quot* of my two *vulgar Fractions* to a *mixt* number, and continuing my *Diviſion* to a *Decimal.*

```
364)1404(3.857142,
     1092
     ----
     3120
     2912
     ----
      2080
      1820
      ----
       2600
       2548
       ----
         520
         364
         ---
         1560
         1456
         ----
          1040
           728
           ---
           312
           ---
```

Here the *Quot* comes out the ſame as in the *decimal* operation, and the *Remainder* 312 being the ſame with the *Remainder* of the firſt *Dividual* renews the *Circle*.

If your *Diviſor* is an *Infinite*, repeting the ſame figure, as .111, or 333, &c. follow the directions laid down in *Diviſion* of *infinite repeting Decimals*, *pages* 196, 197 and 198.

EXAMPLE I.

$.333)82.32,142857, =\frac{1}{3})\ 82\frac{9}{28} =\frac{1}{3})\ \frac{2305}{28}($
$=\frac{6915}{28} =246.96,428571,4$

Here I multiply by 3, and have my *Quot*.

82.32,142857
3
246.96,428571,

EXAMPLE II.

$.666)76.10,714285\ (=\frac{2}{3})\ 76\frac{3}{28}\ (=\frac{2}{3})\ \frac{2131}{28}($
$=\frac{6393}{56})\ =114.160,714285,$

Here I multiply by 3, and divide by 2, and have my *Quot*.

76.10,714285
3
2)228.32,142857
114.160,714285,7

Or I multiply by 1.5=$\frac{3}{2}$

76.10,714285 *Dividend.*
1.5

3805 3,571428
7610,7142857

114.1 60,714285 *Quot.*

EXAMPLE III.

$\frac{6.10,714285}{.555} = \frac{5}{9})\ 6\frac{3}{28} = \frac{5}{9})\ \frac{171}{28}\ (= \frac{1539}{140}$ =10.99,285714,

Here I multiply by 9, and divide by 5, and have my *Quot.*

6,10,714285 *Dividend.*
9

5)54.96,428571
10.99,285714,28 *Quot.*

Or I multiply by 1.8=$\frac{9}{5}$

6.10,714285 *Dividend.*
1.8

488 5,714285
610,7142857

10.99,2 85714,2 *Quot.*

The three laſt *Examples* are ſo manifeſt, both as to the practice and theory, that I think it needleſs to advance any proof, by reducing the *Quot*

Quot of the *vulgar Fractions* to a *mixt Decimal.*

If the *Diviſor* is *infinite*, whether *repeting* or *circulating*, and has a *finite* part, reduce that *Diviſor* to a *finite vulgar Fraction*, by *Rule* II. of *Reduction* of *Circulates*, and by it divide the *Dividend*, whatever it be, provided the circles of both *Dividend* and *Diviſor* conſiſt of an equal number of figures, and you have the *Quot* a *vulgar Fraction*, which, if you pleaſe, you may reduce to a *Decimal.*

EXAMPLE I.

A *mixt* number having an *infinite Repeter*, dividing a *mixt* number having an *infinite Circulate.*

$\frac{76.0,592}{4.53} = 4\frac{8}{15})\ 76\frac{24}{405}(= \frac{68}{15})\ \frac{30804}{405} = \frac{462060}{27540}$
$= 16.777$

$\frac{76.0,592}{4.53} = \frac{408}{90})76.0,592$

```
                 90                450
                                    45
          ———————————             ———
   408)   6845.3333(16.77          405
          408 ' '                    3
          ———                       ———
          2765                     408 num.
          2448
          ————
           3173
           2856
           ————
            3173
            2856
            ————
            317
```

Here, I, firſt of all, reduce my *Diviſor* 4.53, to a *finite vulgar Fraction* $4\frac{08}{90}$, as in the preceeding page, then I divide my *Dividend* 76.0592, by that *finite* Fraction, *viz.* I multiply the *Dividend* by the *Denominator* 90, and divide the *Product* 6845.3333 by the *Numerator* 408, and have the *Quot* 16.777.

As a proof of my work, I ſhall reduce the *Quot* of my *vulgar Fractions*, *viz.* $4\frac{62060}{27540}$, to a *Decimal.*

Here the *Remainder*, 21420, being ſtill the ſame, repetes the ſame figure (7) in the *Quot.*

```
27540)462060,(16.7
      27540
      ------
      186660
      165240
      ------
       214200
       192780
       ------
        21420
        ------
```

E X A M P L E II.

A *mixt* number, conſiſting of an *Integer* and *Circulate*, dividing a *mixt* number conſiſting of an *Integer* and *Circulate*, where both circles have an equal number of figures.

$\frac{500.45,45}{6.36,36}=6\frac{4}{11})\ 500\frac{5}{11}\ (=\frac{70}{11})\frac{5505}{11}\ (=\frac{5505}{70}$
$=78.6,428571$

Here

600
6
―――
594
36
―――
630 *num.*

$6\frac{30}{99}$) 500.45
99
―――――
4504,09
4504,090
―――――
630)49544.99(78.6,428571,
4410
―――
5444
5040
―――
4049
3780
―――
2699
2520
―――
1799
1260
―――
5399
5040
―――
3599
3150
―――
4499
4410
―――
899
630
―――
269
―――

Here I firſt reduce my *Diviſor* 6.36 to a *finite vulgar Fraction* $6\frac{30}{99}$, as in the margin; then I divide my *Dividend*, 500.45, by that Fraction, as in the former *Example*, and find the *Quot* 78.6,428571.

As the procedure here is the ſame as in the former *Example*, I think it needleſs to adduce a proof of my work, by reducing $\frac{5505}{70}$, the *Quot* of my *vulgar Fractions*, to a *Decimal*.

The reaſon of reducing the *Diviſor* only to a *finite* Fraction, when the number of figures in both circles is the ſame, is, the number of figures in both circles being the ſame, the number of Nines in the *Denominators* of both *finite* Fractions is the ſame; but multiplying the given *Dividend* by the *Denominator* of the *finite* part of the *Diviſor*, reduces both to a common *Denominator*; ſo I, in concluſion, divide the *Numerators* only.

If both *Diviſor* and *Dividend* are *Circulates*, whoſe circles do not conſiſt of an equal number of figures, reduce both to their *finite* values, as directed in *Reduction* of *Circulates*, *Rule* II. and divide as in *Vulgar Fractions*; only remember, that, in dividing your *finite* Fractions, the *Denominators* (multiplying the *Numerators* croſs-ways) make a Fraction, whoſe *Numerator* is the *Denominator* of the *Diviſor*, &c. This Fraction, for the greater eaſe and compend in working, you may reduce to its loweſt terms.

Thus, dividing 650.882154 by 8.296, I, firſt of all, reduce both to their *finite* values, as in the margin, and then divide as in *Vulgar Fractions*; only, inſtead of multiplying croſs-ways by the *Denominators*, I firſt reduce the Fraction ariſing from the *Denominators* to its loweſt terms, and multiply by that new Fraction $\frac{1}{1001}=\frac{999}{999999}$; ſo the work will ſtand thus,

$$\begin{array}{rl} 650000000 & \\ 650 & \\ \hline 649999350 & \\ 882154 & \\ \hline 650881504 & \textit{num.} \\ \hline 8000 & \\ 8 & \\ \hline 7992 & \\ 296 & \\ \hline 8288 & \textit{num.} \\ \hline \end{array}$$

$$\frac{650.882154}{8.296}=\frac{8288}{999}\Big)\frac{650881504}{999999}=$$

$$\frac{650881504}{8288}\times\frac{999}{999999}=\frac{650881504}{8288}\times\frac{1}{1001}=$$

$$\frac{650881504}{8296288}=78.45$$

Here, inſtead of multiplying $\frac{650881504}{8288}$ by $\frac{999}{999999}$, I multiply it by $\frac{1}{1001}$, the ſame Fraction in its loweſt terms, and find my *Quot* $\frac{650881504}{8296288}$, which, reduced to a *Decimal*, is 78.45,45

$$\begin{array}{r} 8288 \\ 1001 \\ \hline 8288 \\ 8288 \\ \hline 8296288 \end{array}$$

Here

```
8296288)650.881504,(78.45
        58074016
        ————————
         70141344
         66370304
         ————————
          37710400
          33185152
          ————————
           45252480
           41481440
           ————————
            3771040
           ————————
```

Here the *Remainder*, 3771040, being the ſame with that of the ſecond *Dividual*, renews the circle.

As the *Dividend* $650.882154=650\frac{262}{297}$, and the *Diviſor* $8.296=8\frac{8}{27}$, we ſhall, for a proof of our work, divide $650\frac{262}{297}$ by $8\frac{8}{27}$, and reduce the *Quot* to a *mixt Decimal*. The procedure ſtands thus.

$8\frac{8}{27})650\frac{262}{297}(=\frac{224}{27})\frac{193312}{297}(=\frac{5219424}{66528}$ *Quot.*

Here

```
66528)5219424,(78.45
      465696
      ------
       562464
       532224
       ------
        302400
        266112
        ------
         362880
         332640
         ------
          30240
          ------
```

Here the *Quot* comes out the ſame as in my *decimal Diviſion.*

To conclude my Arithmetick of *Circulates*, I muſt warn my reader, that, as it often happens, that in *Diviſion* of *Circulates*, the *Quot* runs to a great number of *decimal* places before it comes to a circle, he may limit the *Quot* at ſeven or eight *decimal* places without any ſenſible loſs, as is plain from *Obſerve* 2. *page* 180.

Having finiſhed my Arithmetick of *Fractions*, both *vulgar* and *decimal*, I, firſt of all, take Notice that the *Rules* for *Practice* (which I laid down immediately after the doctrine of *vulgar Fractions*, and whoſe number and variety render them burdenſome to the memory, may all be reduced to one *ſimple Rule*.

RULE, *Reduce the fractional part of the price to a* Decimal, *and multiply the price thus* decimally *expreſſed by the number of goods.*

EXAMPLE I.

What is the price of 868 *Scots* quarts of Spirits, at 5 *s. ſterl. per* quart?

Here 5 *s.* = .25 *l.* So the *Anſwer* is 217 *l.* as in the margin.

868
.25
4340
1736
217.00

EXAMPLE II.

What is the price of 6548 *lb.* of Coffee, at 6 *s.* 8 *d. per lb.?*

6 *s.* 8 *d.* = .333 *l.* which multiplied into 6548, gives the *Anſwer* 2182 *l.* 13 *s.* 4 *d.*

6548
0.333
$\frac{1}{3}$=2182.666

EXAMPLE III.

What coſt 56086 yards of tape, at three Farthings *per* yard?

Here 3 *f.* = .003125 *l.* So the *Anſwer* is 175 *l.* 5 *s.* 4 *d.* 2 *f.*

56086
.003125
280430
112172
56086
168258
175.268750

EXAMPLE IV.

What is the price of 276 *Ct.* 2 *qrs.* of Steel, at 2 *l.* 3 *s.* 8 *d.* ½ *per. Ct.*?

Here 2 *l.* 3 *s.* 8 *d.* ½ = 2.185416×276,.5 = 604 *l.* 5 *s.* 4 *d.* ¼

2.185416
276.5
10927083
13112500
1529791666
4370833333
604.2677083

Theſe four *Examples*, I think, are ſufficient to ſhow, that the above *Rule* eaſily performs whatever may be done by the *Rules* of *Practice*.

It is ordinary, after the doctrines of *Vulgar Fractions* and *Decimals*, to give directions for working the *Rules* of *Three*, *Five* and *Fellowſhip*, conformable to theſe two doctrines; but as theſe doctrines are already laid down, the method of ſtating and working the ſeveral queſtions that may offer in theſe *Rules*, clearly ſhown, and a great many queſtions for exerciſing the reader in the *Rules*, are ſubjoined to them, I ſhall only adviſe my reader to reſolve theſe queſtions both *decimally* and by *vulgar Fractions*, and the two following queſtions, which I add for his further exerciſe in *circulating Decimals*.

Queſtion 1. If 25 men do a piece of work in eight weeks four days, how many men will do that work in three weeks and three days?

Anſwer, 62.5=62½ men; that is, 62 mens daily work, and ½ of a man's daily work.

Here the week being the *Integer*, the days, which are 7th parts, come out *Circulates*.

Queſt. 2. If 72 hundreds weight, 2 quarters and 4 pound, coſt 157 *l.* 4 *s.* what will 168 hundreds and 8 pound coſt?

Anſ. 364 *l.* 4 *s.* 11 *d.*

Here, for the reader's conveniency, I ſhall give a *decimal* Table of *Averdupois* weight, for quarters, pounds and ounces, the hundred weight, or 112 *lb.* being the *Integer*.

The

The TABLE.

N.	*qrs.*	*Pounds.*	*Ounces.*
1	.25	0089,285714	.00055803,571428
2	.5	.017,857142	.0011160,714285
3	.75	.0267,857142	.00167410,714285
4		.03,571428	.002232,142857
5		.0446,428571	.00279017,857142
6		.053,571428	.0033482,142857
7		.0625	.00390625
8		.07,142857	.004464,285714
9		.0803,571428	.00502232,142857
10		.089,285714	.00558035,714285
11		.0982,142857	.00613839,285714
12		.107,142857	.006696,428571
13		.1160,714285	.00725446,428571
14		.125	.0078125
15		.1339,285714	.00837053,571428
16		.1,428571	.0089,285714
17		.1517,857142	
18		.160,714285	
19		.1696,428571	
20		.17,857142	
21		.1875	
22		.196,428571	
23		.2053,571428	
24		.2,142857	
25		.2232,142857	
26		.232,142857	
27		.2410,714285	
28		.25	

The uſe of the preceeding *Table.*

Find the number of *Quarters*, *Pounds* or *Ounces*, in the left hand column, and oppoſite to it, and directly below the Words *Quarters*, *Pounds*, *Ounces*, you have the *Decimal.*

If you would have the *Decimal* of *Quarters*, *Pounds* and *Ounces*, take their ſeparate *Decimals* from the *Table*, and add them thus;

What is the *Decimal* of three *Quarters*, twelve *Pounds* and nine *Ounces?*

3 *qrs.* = .75
12 *lb.* = .10714285,714285
9 *oz.* = .00502232,142857

---

3 *qrs.* 12 *lb.* 9 *oz.* = .86216517,857142 *C.*

N. B. Through the whole of this *Table* I have always completed one *Circle.*

APPEN-

APPENDIX.

TO conclude this TREATISE, I ſhall here lay down the practice of *croſs Multiplication*, as it is commonly made uſe of in the meaſure of *Maſon* and *Carpenter* work.

As the *French* divide their inch into twelve lines, ſo our maſons, in meaſuring their work, ſuppoſe the inch divided into twelve equal parts, which we ſhall likewiſe call lines: So the meaſure, upon which our Calcul depends, ſtands thus.

Lineal, or *length-meaſure*.

12 lines = 1 inch.
12 inches = 1 foot.
3 feet = 1 yard.
6 yards = 1 rood.

Superficial meaſure.

144 lines = 1 inch.
144 inches = 1 foot.
9 feet = 1 yard.
36 yards = 1 rood.

Solid meaſure.

1728 lines = 1 inch.
1728 inches = 1 foot.
27 feet = 1 yard.
216 yards = 1 rood.

The *ſuperficial content* of any *rectangular quadrilateral* figure, is ſound by multiplying the length by the height or breadth; and that of a *right-angled Triangle* is ſound by multiplying the baſe by ½ the height.

Solid meaſure is ſound by multiplying the ſurface by the thickneſs.

The lengths, breadths and thickneſſes, are taken in lineal feet, inches and lines; ſo the whole calcul of the *contents* is performed by a *croſs Multiplication* of feet, inches and lines, by feet, inches and lines.

EXAMPLE I.

In an area, paved with free ſtone, in length 22 feet 4 inches, and in breadth 19 feet 7 inches, how many ſquare feet?

Anſwer, 437 feet, 4 inches, 4 lines.

F.	*in.*	*l.*
22	4	0
19	7	0
418	0	0
6	4	0
12	10	0
	2	4
437	4	4

Here I multiply my 22 feet in the *Multiplicand*, by 19 feet in the *Multiplier*, and have 418 ſquare feet; then I multiply 19 feet into 4 inches in the *Multiplicand*, the *Product*, 76, affords 6 twelves, or 6 feet and 4 inches.

Next, I multiply 7 inches in the *Multiplier* into 22 feet in the *Multiplicand*, the *Product*, 154, affords 12 feet, which I ſet down under feet, and the remaining 10 inches I ſet under inches; then I multiply 7 inches into 4 inches, the *Product*, 28, gives 2 inches, which I ſet down under inches, and the remaining 4 I ſet in the place of lines; theſe added, give the *Anſwer* 437 feet, 4 inches, 4 lines.

If I would reduce this *Product* to yards, I divide my 437 feet by 9, the number of ſquare feet in a ſquare yard, as in the margin, and have my area, 48 yards, 5 feet, 4 inches, 4 lines.

	F.	in.	l.
	9)437	4	4
yds.	*f.*	*in.*	*l.*
48	5	4	4

If I would reduce theſe 48 yards to roods, I divide by 36, the number of yards in a rood, and find my area to be 1 rood, 12 yards, 5 feet, 4 inches, 4 lines.

36)48(1 *rood.*
36
—
12 *yards.*

EXAMPLE II.

In a floor 49 feet, 7 inches, 4 lines long, and 26 feet 6 inches broad, how many ſquare feet?

Anſwer, 1314 feet, 8 inches, 4 lines.

Here, as in the former *Example*, I firſt multiply my 26 feet into the whole *Multiplicand*, and then my 6 inches into the whole *Multiplicand*.

In multiplying my 26 feet into 4 lines, the *Product*, 104, affords 8 twelves, or 8 inches, and 8 lines, and in multiplying my 6 inches into 4 lines, the *Product*, 24, affords juſt 2 lines.

F.	in.	l.
49	7	4
26	6	0
1274	0	0
15	2	0
0	8	8
24	6	0
	3	6
		2
1314	8	4

EXAMPLE III.

In a piece of timber, whoſe length is 17 feet 6 inches, breadth 1 foot 11 inches, and thickneſs 2 feet 7 inches, how many ſolid feet?

Anſwer, 86 feet, 7 inches, 9 lines $\frac{1}{2}$.

Here I firſt multiply my length, 17 feet, 6 inches, by 1 foot, 11 inches of breadth, and find the *ſuperficial* meaſure of the baſe of my *ſolid* to be 33 feet, 6 inches, 6 lines, which I multiply by the thickneſs, 2 feet 7 inches, and find the *ſolid content* to be 86 feet, 7 inches, 9 lines; for the Fraction of the line may be neglected.

F.	*in.*	*l.*	
17	6	0	
1	11	0	
17	6	0	
15	7	0	
	5	6	
33	6	6	
2	7		
66	0	0	
1	0	0	
0	1	0	
19	3	0	
00	3	6	
00	0	3	6
86	7	9	6

Let theſe three *Examples* ſuffice to illuſtrate and explain the uſe and practice of *croſs Multiplication* in the menſuration of *Surfaces* and *Solids.* Mean time, I could heartily wiſh, that theſe Gentlemen, who have daily occaſion for that kind of meaſure, would rather chooſe the *decimal* way of working, as being fully as expeditious, and leſs liable to miſtakes, eſpecially conſidering in our rulers of foot-meaſure the inch is divided, not into twelve, but ten equal parts, which naturally introduces the *decimal* way. For their greater eaſe and conveniency in working, I ſhall hereto ſubjoin two *decimal* tables, one of inches, a foot being the *Integer*, another of feet and inches, a yard being the *Integer*.

If there are ten parts of an inch in the meaſure, theſe Tenths may be reduced to the *Decimal* of a foot, by removing the *decimal* point of the like number of inches one place further to the left hand.

Thus, if it is demanded, what *Decimal* of a foot anſwers to $\frac{5}{10}$ of an inch, I find in the Table, that 5 inches = .4166 of a foot; then $\frac{5}{10}$ of an inch = .04166 of a foot.

If it is demanded, what *Decimal* of a foot anſwers to 7 inches, and $\frac{5}{10}$ of an inch, to the *Decimal*, anſwering to 7 inches, add the *Decimal* anſwering to $\frac{5}{10}$ of an inch.

7 inches = .58333 of a foot.
$\frac{5}{10}$ of an inch = .04166 of a foot.

---

7 inches $\frac{5}{10}$ = .625

Table of inches, *a lineal foot the* Integer.

N.	*inches.*
1	.0833
2	.1666
3	.25
4	.3333
5	.4166
6	.5
7	.5833
8	.6666
9	.75
10	.8333
11	.9166
12	1.0000

In this Table of feet and inches, ſeek the feet in the left hand column, and the inches in the next column, and oppoſite to inches you have the *Decimal.*

Thus 2 feet 5 inches =.8055

The *Decimal* of a yard anſwering to any number of 10th parts of an inch, is found the ſame way as in the preceeding Table.

Table of Feet *and* Inches, *a yard being the* Integer.

Feet.	*Inches.*	*Decimal parts.*
	1	.0277
	2	.0555
	3	.0833
	4	.1111
	5	.1388
	6	.1666
	7	.1944
	8	.2222
	9	.25
	10	.2777
	11	.3055
i	12	.3333
	1	.3611
	2	.3888
	3	.4166
	4	.4444
	5	.4722
	6	.5
	7	.5277
	8	.5555
	9	.5833
	10	.6111
	11	.6388
ii	12	.6666
	1	.6944
	2	.7222
	3	.75
	4	.7777
	5	.8055
	6	.8333
	7	.8611
	8	.8888
	9	.9166
	10	.9444
	11	.9722
iii	12	1.0000

I ſhall here add a few *Examples* of the calcul of ſuperficial and ſolid meaſure by *Decimals.*

EXAMPLE I.

What is the area of a floor 25 feet 7 inches in length, and 17 feet 5 inches in breadth?

F. *in.*
25 7 = 25.583
17 5 = 17.4166

$\frac{1}{3}$ = 85 2766
$\frac{1}{3}$ = 85 2766
2558333
10233 3333
17908 3 3333
25583 3 3333

445.5763 866

The *Anſwer* comes out 445 ſquare feet, and .576386 of a ſquare foot, which (by *Rule* III. of *Reduction* of *Decimals*) multiplied by 144, the number of ſquare inches in a foot, gives 82 ſquare inches, and .99968 of a ſquare inch.

.576386
144

2305546
23055466
57638666

82.99968 *inches.*

EXAMPLE II.

In a room 4 yards, 4 inches high, and 28 yards, 11 inches about, how many ſquare yards?

Here I multiply according to the *Rules* of *Infinite repeting Decimals* (both *Factors* being *Repeters*) and find 116 ſquare yards, and .367, which multiplied by 9, the number of ſquare feet in a yard, gives 3 ſquare feet and .310 of a foot, which multiplied by 144, give 44 ſquare inches, and .64 of an inch: So the *Anſwer* is 116 yards, 3 feet, 44 inches, and .64 of an inch.

```
 Y.   in.
 28   11=  28.305
  4    4=      4.111
---------------------
     1/9=   3 145
          113 222
          -------
          116.367 sq. yds.
                9
          --------
            3.310 feet.
              144
            -----
            1 240
           1240
           310
           ------
           44.640 inches.
```

EXAMPLE III.

What number of yards will take up that room whoſe height is 4 yards, 3 inches, and compaſs 34 yards, 2 feet, and 6.3 inches?

```
F.  in.                Y.  f.  in.
2   6   =.8333         34  2   6.3=34.8416
0   0.3=.0083           4  0   3  =    4.083
---------------        ----------------------
2   6.3=.8416                 1/3 =11 61 388
---------------                   278 73 333
Anſ. 142 y. 2 f.                 1393 66 666
52.1 in.                         -----------
                                 142.2701 388
```

E X-

EXAMPLE IV.

How many ſolid yards of digging in a cellar that is 8 yards, 1 foot, 4 inches long, 5 yards 8 inches broad, and 2 yards, 1 foot, 4 inches deep?

Anſ. 107.796982 cubic yards.

Y. f. in.
8 1 4 = 8.444
5 0 8 = 5.222

$\frac{1}{9}$ = 938 2716
2

18765432
42222 2222

440987654

2 1 4 = 2.444

$\frac{1}{9}$ = 4899 8628
4

195994512
881975308

107.796982 *yards.*
27

5578874
1593964

21.518514 *feet.*
1728

4148112
1037028
3629598
518514

895.992192 *in.*

The *Decimal* .796982 I multiply by 27 cubic feet in a cubic yard, the *Product* gives 21.518514 feet; the *Decimal*, .518514, I multiply by 1728 cubic inches in a cubic foot, I find 895.992192 cubic inches: So the *Anſwer* in all comes to 107 yards, 21 feet, 895.992192 inches.

To conclude, I muſt here take notice, that, in working *decimally*, we have real ſolid inches, or 1728th parts of a ſolid foot; and in ſuperficial meaſure we have real ſquare inches, or 144th parts of a ſquare foot; whereas, in working by croſs *Multiplication*, we have only 12th parts of a ſolid foot, which we call inches, and only 12th parts of a ſquare foot, which we call ſquare inches.

FINIS.